PUTNAM EXAM SEMINAR
FALL 2012

QUIZ 3
SEPTEMBER 19

Problem 1. If α, β and γ are the interior angles of a triangle, show that[1]

$$\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}\leq\frac{1}{4}.$$

Problem 2. A 2×3 rectangle has vertices at $(0,0)$, $(3,0)$, $(3,2)$ and $(0,2)$. If it is rotated 90° clockwise about the point $(3,0)$, determine the distance traveled by the point whose initial position is $(1,1)$. [Putnam 1991, A1]

Problem 3. A goat is tethered with a rope of length $20\sqrt{3}$ feet to the center of a fenced square field with sides of length 60 feet. Determine the area of the region over which the goat can graze.

Problem 4. A regular pentagon is inscribed inside a unit circle. If A_0, A_1, A_2, A_3 and A_4 denote its vertices in clockwise order, show that

$$|A_0A_1||A_0A_2|=\sqrt{5}.$$

[1]The stated inequality follows from geometric principles alone. However, if one is willing to use calculus, the 1/4 can be replaced with 1/8.