Problem 1. Evaluate

$$\int_0^{2\pi} \frac{dx}{1 + e^{\sin x}}.$$

Problem 2. Let $p(x) = 2 + 4x + 3x^2 + 5x^3 + 3x^4 + 4x^5 + 2x^6$. For $0 < k < 5$ define

$$I_k = \int_0^\infty \frac{x^k}{p(x)} \, dx.$$

For which k is I_k smallest?

Problem 3. Find all real-valued continuously differentiable functions f defined on the real line such that for all x

$$(f(x))^2 = 1990 + \int_0^x \left[(f(t))^2 + (f'(t))^2 \right] dt.$$

[Putnam 1990, B1]

Problem 4. Evaluate

$$\int_0^a \int_0^b e^{\max\{b^2 x^2, a^2 y^2\}} \, dx \, dy,$$

where a and b are positive. [Putnam 1989, A2]

Problem 5. Find all continuous, positive functions f on $[0, 1]$ so that

$$\int_0^1 f(x) \, dx = 1,$$

$$\int_0^1 x f(x) \, dx = \alpha,$$

$$\int_0^1 x^2 f(x) \, dx = \alpha^2,$$

where α is a given real number. [Putnam 1964, 2]