PUTNAM EXAM SEMINAR
FALL 2012

QUIZ 6
OCTOBER10

Problem 1. Assume that x, y and z are positive real numbers that satisfy the system of equations

$$\begin{aligned} x + y + xy &= 8, \\ y + z + yz &= 15, \\ z + x + xz &= 35. \end{aligned}$$

Determine the value of $x + y + z + xyz$.

Problem 2. Find all sets of four real numbers such that the sum of any one and the product of the other three is always equal to 2.

Problem 3. Prove that there are only a finite number of ordered triples $T = (x - y, y - z, z - x)$, where x, y and z are complex numbers satisfying

$$x(x-1) + 2yz = y(y-1) + 2zx = z(z-1) + 2xy,$$

and list all such triples T. [Putnam 1986, B2]

Problem 4. Find all positive integers n, $k_1, \cdots, k_n$ such that

$$\begin{aligned} k_1 + \cdots + k_n &= 5n - 4, \\ \frac{1}{k_1} + \cdots + \frac{1}{k_n} &= 1. \end{aligned}$$

[Putnam 2005, B2]