



PUTNAM EXAM SEMINAR
FALL 2012

QUIZ 8
OCTOBER 24

Problem 1. Basketball star Shanille O'Keal's team statistician keeps track of the number, $S(N)$, of successful free throw attempts she has made in her first N attempts of the season. Early in the season, $S(N)$ was less than 80% of N , but by the end of the season $S(N)$ was more than 80% of N . Was there necessarily a moment when $S(N)$ was exactly 80% of N ? [Putnam 2004, A1]

Problem 2. Let k be a positive integer. Suppose that the integers $1, 2, \dots, 3k + 1$ are written down in random order. What is the probability that at no time during this process, the sum of the integers that have been written up to that time is a positive integer divisible by 3? Your answer should be in closed form, but may include factorials. [Putnam 2007, A3]

Problem 3. An unbiased coin is tossed n times. Find a formula, in closed form, for the expected value of $|H - T|$, where H is the number of heads, and T is the number of tails. [Putnam 1974, A4 (modified)]

Problem 4. Let v and w be distinct, randomly chosen roots of the equation $z^{2012} - 1 = 0$. Find the probability that $\sqrt{2 + \sqrt{3}} \leq |v + w|$. [American Invitational Math. Exam, 1997 (modified)]