Problem 1. Determine the minimum value of

$$(r-1)^2 + \left(\frac{s}{r}-1\right)^2 + \left(\frac{t}{s}-1\right)^2 + \left(\frac{4}{t}-1\right)^2$$

if r, s and t are real numbers with $1 \le r \le s \le t \le 4$.

Problem 2. Find the minimum value of

$$|\sin x + \cos x + \tan x + \cot x + \sec x + \csc x|$$

for real numbers x. [Putnam 2003, A3]

Problem 3. Find the least possible area of a convex set in the plane that intersects both branches of the hyperbola $xy = 1$ and both branches of the hyperbola $xy = -1$. (A set S in the plane is called *convex* if for any two points in S the line segment connecting them is contained in S.) [Putnam 2007, A2]

Problem 4. For each continuous function $f : [0,1] \to \mathbb{R}$, let $I(f) = \int_0^1 x^2 f(x)\, dx$ and $J(x) = \int_0^1 x(f(x))^2\, dx$. Find the maximum value of $I(f) - J(f)$ over all such functions f. [Putnam 2006, B5]