Number Theory II | Assignment 11.1
Fall 2012 | Due November 15

Exercise 1. Let G and H be groups.

a. Show that if $f_1 \in \widehat{G}$ and $f_2 \in \widehat{H}$, and we define $(f_1 \otimes f_2)(x,y) = f_1(x)f_2(y)$, then $f_1 \otimes f_2 \in \widehat{G \times H}$.

b. Recall that given $f \in \widehat{G \times H}$, we defined $f_G(x) = f(x,e)$ and $f_H(y) = f(e,y)$. Show that $(f_G, f_H) \in \widehat{G} \times \widehat{H}$.

c. Show that the functions given by $\rho(f) = (f_G, f_H)$ and $\sigma(f_1, f_2) = f_1 \otimes f_2$ are inverses.

Exercise 2. Show that the map $\alpha : \widehat{\mathbb{Z}} \to \mathbb{C}^\times$ given by $\alpha(f) = f(1)$ is an isomorphism. Conclude that $|\widehat{\mathbb{Z}}| \neq |\mathbb{Z}|$.