Number Theory II
Fall 2012

Assignment 8.2
Due October 23

Exercise 1. Let a be a nonnegative real number and f be a function that is Riemann integrable on every finite subinterval of $[a, \infty)$. Let

$$\overline{f}(x) = \frac{1}{x} \int_a^x f(t)\, dt$$

for $x > a$. Suppose that $\lim_{x \to \infty} f(x) = 0$.

a. Let $\epsilon > 0$ and choose $x_0 > a$ so that $|f(t)| < \epsilon/2$ for $t \geq x_0$. By splitting the integral at x_0, show that

$$\left|\overline{f}(x)\right| \leq \frac{\epsilon}{2} + \frac{C(x_0)}{x}$$

for $x \geq x_0$, where $C(x_0)$ is a constant that may depend on x_0.

b. Use part **a** to show that $\lim_{x \to \infty} \overline{f}(x) = 0$.

Exercise 2. Show that the converse to the result obtained in the preceding exercise is false. That is, $\lim_{x \to \infty} \overline{f}(x) = 0$ need not imply that $\lim_{x \to \infty} f(x) = 0$.

Exercise 3. Prove that if $\dfrac{\pi(x)}{x/\log x} = O(1)$, then $\dfrac{\psi(x)}{x} = O(1)$. If the constant in the first big-oh is made explicit (e.g. as in Theorem 4.6), what can you say about the constant in the second?