



NUMBER THEORY II  
FALL 2012

ASSIGNMENT 9  
DUE NOVEMBER 1

**Exercise 1.** Let  $\epsilon > 0$ . Show that the prime number theorem implies that

$$\prod_{p \leq x} p \leq e^{(1+\epsilon)x}$$

for all sufficiently large  $x$ . How does this compare to the bound we proved in class?

**Exercise 2.** Recall that the *logarithmic integral function* is defined for  $x \geq 2$  by

$$\text{Li}(x) = \int_2^x \frac{dt}{\log t}.$$

Show that

$$\lim_{x \rightarrow \infty} \frac{\text{Li}(x)}{x/\log x} = 1.$$

[*Suggestion:* Use the results of exercise 4.19.]

**Exercise 3.** Exercise 4.16.