



PUTNAM EXAM SEMINAR  
FALL 2013

AUGUST 28

**Exercise.** A *triangular number* is a positive integer of the form  $n(n+1)/2$ . Show that an integer  $m$  is the sum of two triangular numbers if and only if  $4m+1$  is the sum of two squares. [Putnam 1975, A1]

**Exercise.** Find polynomials  $a(x)$ ,  $b(x)$ ,  $c(x)$  such that

$$|a(x)| - |b(x)| + c(x) = \begin{cases} -1 & \text{if } x < -1, \\ 3x + 2 & \text{if } -1 \leq x \leq 0, \\ -2x + 2 & \text{if } x > 0. \end{cases}$$

[Putnam 1999, A1]