



PUTNAM EXAM SEMINAR
FALL 2013

ASSIGNMENT 7
DUE OCTOBER 28

Exercise 1. Evaluate

$$\int_0^\infty \left(x - \frac{x^3}{2} + \frac{x^5}{2 \cdot 4} - \frac{x^7}{2 \cdot 4 \cdot 6} + \cdots \right) \left(1 + \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} + \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \cdots \right) dx.$$

[Putnam 1997, A3]

Exercise 2. Sum the series

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m^2 n}{3^m (n3^m + m3^n)}.$$

[Putnam 1999, A4]

Exercise 3. Let f be an infinitely differentiable real-valued function defined on the real numbers. If

$$f\left(\frac{1}{n}\right) = \frac{n^2}{n^2 + 1}, \quad n = 1, 2, 3, \dots,$$

compute the values of the derivatives $f^{(k)}(0)$, $k = 1, 2, 3, \dots$ [Putnam 1992, A4]