PUTNAM EXAM SEMINAR
FALL 2013

ASSIGNMENT 9
DUE NOVEMBER 18

Exercise 1. Find the least number A such that for any two squares of combined area 1, a rectangle of area A exists such that the two squares can be packed in the rectangle (without interior overlap). You may assume that the sides of the squares are parallel to the sides of the rectangle. [Putnam 2003, A3]

Exercise 2. Determine the minimum value of

$$(r-1)^2 + \left(\frac{s}{r}-1\right)^2 + \left(\frac{t}{s}-1\right)^2 + \left(\frac{4}{t}-1\right)^2$$

if r, s and t are real numbers with $1 \le r \le s \le t \le 4$.

Exercise 3. Find the least possible area of a convex set in the plane that intersects both branches of the hyperbola $xy = 1$ and both branches of the hyperbola $xy = -1$. (A set S in the plane is called *convex* if for any two points in S, the line segment connecting them is contained in S.) [Putnam 2007, A2]