



PUTNAM EXAM SEMINAR
FALL 2013

QUIZ 2
SEPTEMBER 18

Exercise 1. The sequence $\{a_n\}_{n=1}^{\infty}$ is defined by $a_1 = 1$, $a_2 = 2$, $a_3 = 24$ and, for $n \geq 4$,

$$a_n = \frac{6a_{n-1}^2 a_{n-3} - 8a_{n-1} a_{n-2}^2}{a_{n-2} a_{n-3}}.$$

Show that a_n is an integer for all n . [Putnam 1999, A6, modified: The original problem asks you to show that, in addition, a_n is always divisible by n]

Exercise 2. Let $\{x_n\}_{n=0}^{\infty}$ be a sequence of nonzero real numbers such that $x_n^2 - x_{n-1}x_{n+1} = 1$ for $n \geq 1$. Prove that there is a real number a so that $x_{n+1} = ax_n - x_{n-1}$ for all $n \geq 1$. [Putnam 1993, A2]

Exercise 3. Show that every positive integer can be written as the sum of integers of the form $2^s 3^t$, such that no summand divides another.