



PUTNAM EXAM SEMINAR
FALL 2013

QUIZ 5
OCTOBER 23

Exercise 1. For all real x , the real-valued function $y = f(x)$ satisfies

$$y'' - 2y' + y = 2e^x.$$

- a. If $f(x) > 0$ for all real x , must $f'(x) > 0$ for all real x ?
- b. If $f'(x) > 0$ for all real x , must $f(x) > 0$ for all real x ?

[Putnam 1987, A3]

Exercise 2. Functions f, g and h are differentiable on some open interval around 0 and satisfy the equations and initial conditions

$$\begin{aligned}f' &= 2f^2gh + \frac{1}{gh}, & f(0) &= 1, \\g' &= fg^2h + \frac{4}{fh}, & g(0) &= 1, \\h' &= 3fgh^2 + \frac{1}{fg}, & h(0) &= 1.\end{aligned}$$

Find an explicit formula for $f(x)$, valid in some open interval around 0. [Putnam 2009, A2]