Putnam Exam Seminar — Quiz 7
Fall 2013 — November 13

Exercise 1. Let G be a finite group of order n with identity element e. If $a_1, a_2, \ldots, a_n$ are n elements of G, not necessarily distinct, prove that there are integers p and q with $1 \leq p \leq q \leq n$ such that $a_p a_{p+1} \cdots a_q = e$.

Exercise 2. Let $*$ be a commutative and associative binary operation on a set S. Assume that for every x and y in S, there exists z in S such that $x * z = y$ (this z may depend on x and y). Show that if a, b, c are in S and $a * c = b * c$, then $a = b$. [Putnam 2012, A2]

Exercise 3. Let G be a group with identity e and $\phi : G \to G$ a function such that

$$\phi(g_1)\phi(g_2)\phi(g_3) = \phi(h_1)\phi(h_2)\phi(h_3)$$

whenever $g_1 g_2 g_3 = h_1 h_2 h_3$. Prove that there exists an element $a \in G$ such that $\psi(x) = a\phi(x)$ is a homomorphism (i.e. $\psi(xy) = \psi(x)\psi(y)$ for all $x, y \in G$). [Putnam 1997, A4]