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$\frac{1}{3} \cdot \text{Eq.2} \mapsto \text{Eq.2}$

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$\frac{1}{3} \cdot \text{Eq.2} \mapsto \text{Eq.2}$ $\qquad$ $\frac{1}{3} \cdot \text{Row2} \mapsto \text{Row2}$

$$\begin{array}{rcrcr} x_1 & + & 5x_2 & = & 7 \\ & & x_2 & = & 3 \end{array} \qquad \left(\begin{array}{ccc} 1 & 5 & 7 \\ 0 & 1 & 3 \end{array} \right)$$

$$-5 \cdot \text{Eq.2} + \text{Eq.1} \mapsto \text{Eq.1}$$

$-5 \cdot \text{Eq.2} + \text{Eq.1} \mapsto \text{Eq.1}$

$-5 \cdot \text{Row2} + \text{Row1} \mapsto \text{Row1}$

$-5 \cdot \text{Eq.2} + \text{Eq.1} \mapsto \text{Eq.1}$

$-5 \cdot \text{Row2} + \text{Row1} \mapsto \text{Row1}$

$$\begin{aligned} x_1 &= -8 \\ x_2 &= 3 \end{aligned}$$

$-5 \cdot \text{Eq.2} + \text{Eq.1} \mapsto \text{Eq.1}$

$$\begin{aligned} x_1 &= -8 \\ x_2 &= 3 \end{aligned}$$

$-5 \cdot \text{Row2} + \text{Row1} \mapsto \text{Row1}$

$$\begin{pmatrix} 1 & 0 & -8 \\ 0 & 1 & 3 \end{pmatrix}$$

$-5 \cdot \text{Eq.2} + \text{Eq.1} \mapsto \text{Eq.1}$

$$\begin{aligned} x_1 &= -8 \\ x_2 &= 3 \end{aligned}$$

$-5 \cdot \text{Row2} + \text{Row1} \mapsto \text{Row1}$

$$\begin{pmatrix} 1 & 0 & -8 \\ 0 & 1 & 3 \end{pmatrix}$$

So,

$\boxed{(-8, 3) \text{ is the only solution.}}$

Example 2 - Solve the linear system.

$$
\begin{array}{rcrcrcr}
 & & 2x_2 & + & x_3 & = & -8 \\
x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\
-x_1 & + & x_2 & + & 2x_3 & = & 3
\end{array}
$$

Example 2 - Solve the linear system.

$$\begin{array}{rcrcrcr} & & 2x_2 & + & x_3 & = & -8 \\ x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ -x_1 & + & x_2 & + & 2x_3 & = & 3 \end{array} \qquad \left(\begin{array}{cccc} 0 & 2 & 1 & -8 \\ 1 & -2 & -3 & 0 \\ -1 & 1 & 2 & 3 \end{array} \right)$$

Example 2 - Solve the linear system.

$$\begin{array}{rcrcrcr} & & 2x_2 & + & x_3 & = & -8 \\ x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ -x_1 & + & x_2 & + & 2x_3 & = & 3 \end{array} \qquad \left(\begin{array}{cccc} 0 & 2 & 1 & -8 \\ 1 & -2 & -3 & 0 \\ -1 & 1 & 2 & 3 \end{array} \right)$$

Eq.1 $\longleftrightarrow$ Eq.2

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$$\begin{array}{rcrcrcr} & & 2x_2 & + & x_3 & = & -8 \\ x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ -x_1 & + & x_2 & + & 2x_3 & = & 3 \end{array}$$

$$\left(\begin{array}{cccc} 0 & 2 & 1 & -8 \\ 1 & -2 & -3 & 0 \\ -1 & 1 & 2 & 3 \end{array}\right)$$

Eq.1 $\longleftrightarrow$ Eq.2

Row1 $\longleftrightarrow$ Row2

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$$\begin{array}{rcrcrcr} & & 2x_2 & + & x_3 & = & -8 \\ x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ -x_1 & + & x_2 & + & 2x_3 & = & 3 \end{array} \qquad \left(\begin{array}{cccc} 0 & 2 & 1 & -8 \\ 1 & -2 & -3 & 0 \\ -1 & 1 & 2 & 3 \end{array} \right)$$

Eq.1 $\longleftrightarrow$ Eq.2 Row1 $\longleftrightarrow$ Row2

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ & & 2x_2 & + & x_3 & = & -8 \\ -x_1 & + & x_2 & + & 2x_3 & = & 3 \end{array} \qquad \left(\begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ -1 & 1 & 2 & 3 \end{array} \right)$$

Example 2 - Solve the linear system.

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$$\left(\begin{array}{cccc} 0 & 2 & 1 & -8 \\ 1 & -2 & -3 & 0 \\ -1 & 1 & 2 & 3 \end{array}\right)$$

$\text{Eq.1} \longleftrightarrow \text{Eq.2}$

$\text{Row1} \longleftrightarrow \text{Row2}$

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ & & 2x_2 & + & x_3 & = & -8 \\ -x_1 & + & x_2 & + & 2x_3 & = & 3 \end{array}$$

$$\left(\begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ -1 & 1 & 2 & 3 \end{array}\right)$$

$\text{Eq.1} + \text{Eq.3} \mapsto \text{Eq.3}$

$\text{Row1} + \text{Row3} \mapsto \text{Row3}$

$$
\begin{array}{rcrcrcr}
x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\
 & & 2x_2 & + & x_3 & = & -8 \\
 & - & x_2 & - & x_3 & = & 3
\end{array}
\qquad
\left(\begin{array}{cccc}
1 & -2 & -3 & 0 \\
0 & 2 & 1 & -8 \\
0 & -1 & -1 & 3
\end{array}\right)
$$

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ & & 2x_2 & + & x_3 & = & -8 \\ & - & x_2 & - & x_3 & = & 3 \end{array}$$

$$\left(\begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ 0 & -1 & -1 & 3 \end{array}\right)$$

Eq.2 $\longleftrightarrow$ Eq.3

Row2 $\longleftrightarrow$ Row3

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ & & 2x_2 & + & x_3 & = & -8 \\ & - & x_2 & - & x_3 & = & 3 \end{array} \qquad \left(\begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ 0 & -1 & -1 & 3 \end{array} \right)$$

Eq.2 $\longleftrightarrow$ Eq.3 Row2 $\longleftrightarrow$ Row3

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ & - & x_2 & - & x_3 & = & 3 \\ & & 2x_2 & + & x_3 & = & -8 \end{array} \qquad \left(\begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & -1 & -1 & 3 \\ 0 & 2 & 1 & -8 \end{array} \right)$$

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ & & 2x_2 & + & x_3 & = & -8 \\ & - & x_2 & - & x_3 & = & 3 \end{array}$$

$$\begin{pmatrix} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ 0 & -1 & -1 & 3 \end{pmatrix}$$

$\text{Eq.2} \longleftrightarrow \text{Eq.3}$

$\text{Row2} \longleftrightarrow \text{Row3}$

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ & - & x_2 & - & x_3 & = & 3 \\ & & 2x_2 & + & x_3 & = & -8 \end{array}$$

$$\begin{pmatrix} 1 & -2 & -3 & 0 \\ 0 & -1 & -1 & 3 \\ 0 & 2 & 1 & -8 \end{pmatrix}$$

$2 \cdot \text{Eq.2} + \text{Eq.3} \mapsto \text{Eq.3}$

$2 \cdot \text{Row1} + \text{Row3} \mapsto \text{Row3}$

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ & - & x_2 & - & x_3 & = & 3 \\ & & & - & x_3 & = & -2 \end{array}$$

$$\left(\begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & -1 & -1 & 3 \\ 0 & 0 & -1 & -2 \end{array}\right)$$

$-\text{Eq.}3 + \text{Eq.}2 \mapsto \text{Eq.}2$

$-\text{Row}3 + \text{Row}2 \mapsto \text{Row}2$

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ & - & x_2 & - & x_3 & = & 3 \\ & & & - & x_3 & = & -2 \end{array}$$

$$\left(\begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & -1 & -1 & 3 \\ 0 & 0 & -1 & -2 \end{array}\right)$$

$$-\text{Eq.3} + \text{Eq.2} \mapsto \text{Eq.2}$$
$$-3 \cdot \text{Eq.3} + \text{Eq.1} \mapsto \text{Eq.1}$$

$$-\text{Row3} + \text{Row2} \mapsto \text{Row2}$$
$$-3 \cdot \text{Row3} + \text{Row1} \mapsto \text{Row1}$$

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ & - & x_2 & - & x_3 & = & 3 \\ & & & - & x_3 & = & -2 \end{array}$$

$$\left(\begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & -1 & -1 & 3 \\ 0 & 0 & -1 & -2 \end{array}\right)$$

$$-\text{Eq.3} + \text{Eq.2} \mapsto \text{Eq.2}$$
$$-3 \cdot \text{Eq.3} + \text{Eq.1} \mapsto \text{Eq.1}$$

$$-\text{Row3} + \text{Row2} \mapsto \text{Row2}$$
$$-3 \cdot \text{Row3} + \text{Row1} \mapsto \text{Row1}$$

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & & & = & 6 \\ & - & x_2 & & & = & 5 \\ & & & - & x_3 & = & -2 \end{array}$$

$$\left(\begin{array}{cccc} 1 & -2 & 0 & 6 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & -1 & -2 \end{array}\right)$$

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ & - & x_2 & - & x_3 & = & 3 \\ & & & - & x_3 & = & -2 \end{array}$$

$$\begin{pmatrix} 1 & -2 & -3 & 0 \\ 0 & -1 & -1 & 3 \\ 0 & 0 & -1 & -2 \end{pmatrix}$$

$$-\text{Eq.3} + \text{Eq.2} \mapsto \text{Eq.2}$$
$$-3 \cdot \text{Eq.3} + \text{Eq.1} \mapsto \text{Eq.1}$$

$$-\text{Row3} + \text{Row2} \mapsto \text{Row2}$$
$$-3 \cdot \text{Row3} + \text{Row1} \mapsto \text{Row1}$$

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & & & = & 6 \\ & - & x_2 & & & = & 5 \\ & & & - & x_3 & = & -2 \end{array}$$

$$\begin{pmatrix} 1 & -2 & 0 & 6 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & -1 & -2 \end{pmatrix}$$

$$-2\text{Eq.2} + \text{Eq.1} \mapsto \text{Eq.1}$$

$$-2\text{Row2} + \text{Row1} \mapsto \text{Row1}$$

$$
\begin{array}{rcrcrcl}
x_1 & & & & & = & -4 \\
 & - & x_2 & & & = & 5 \\
 & & & - & x_3 & = & -2
\end{array}
\qquad
\left(\begin{array}{cccc}
1 & 0 & 0 & -4 \\
0 & -1 & 0 & 5 \\
0 & 0 & -1 & -2
\end{array}\right)
$$

$$
\begin{array}{rcrcrcl}
x_1 & & & & & = & -4 \\
 & - & x_2 & & & = & 5 \\
 & & & - & x_3 & = & -2
\end{array}
\qquad
\left(\begin{array}{cccc}
1 & 0 & 0 & -4 \\
0 & -1 & 0 & 5 \\
0 & 0 & -1 & -2
\end{array}\right)
$$

$-\text{Eq.2} \mapsto \text{Eq.2}$ $\qquad$ $-\text{Row2} \mapsto \text{Row2}$

$$
\begin{array}{rcrcrcl}
x_1 & & & & & = & -4 \\
 & - & x_2 & & & = & 5 \\
 & & & - & x_3 & = & -2
\end{array}
\qquad
\left(\begin{array}{cccc}
1 & 0 & 0 & -4 \\
0 & -1 & 0 & 5 \\
0 & 0 & -1 & -2
\end{array}\right)
$$

$-\text{Eq.2} \mapsto \text{Eq.2}$

$-\text{Eq.3} \mapsto \text{Eq.3}$

$-\text{Row2} \mapsto \text{Row2}$

$-\text{Row3} \mapsto \text{Row3}$

$$\begin{array}{rrrcr} x_1 & & & = & -4 \\ - & x_2 & & = & 5 \\ & - & x_3 & = & -2 \end{array} \qquad \left(\begin{array}{cccc} 1 & 0 & 0 & -4 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & -1 & -2 \end{array}\right)$$

$-\text{Eq.2} \mapsto \text{Eq.2}$ $\qquad$ $-\text{Row2} \mapsto \text{Row2}$

$-\text{Eq.3} \mapsto \text{Eq.3}$ $\qquad$ $-\text{Row3} \mapsto \text{Row3}$

$$\begin{array}{rrrcr} x_1 & & & = & -4 \\ & x_2 & & = & -5 \\ & & x_3 & = & 2 \end{array} \qquad \left(\begin{array}{cccc} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$$\begin{array}{rcrcrcl} x_1 & & & & & = & -4 \\ & - & x_2 & & & = & 5 \\ & & & - & x_3 & = & -2 \end{array} \qquad \left(\begin{array}{cccc} 1 & 0 & 0 & -4 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & -1 & -2 \end{array}\right)$$

$-\text{Eq.2} \mapsto \text{Eq.2}$ $\qquad$ $-\text{Row2} \mapsto \text{Row2}$

$-\text{Eq.3} \mapsto \text{Eq.3}$ $\qquad$ $-\text{Row3} \mapsto \text{Row3}$

$$\begin{array}{rcrcrcl} x_1 & & & & & = & -4 \\ & & x_2 & & & = & -5 \\ & & & & x_3 & = & 2 \end{array} \qquad \left(\begin{array}{cccc} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

So the only solution is $\boxed{(-4, -5, 2)}$.

Example 3 - Solve the linear system.

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 6x_3 & = & 12 \\ 2x_1 & + & 4x_2 & + & 12x_3 & = & -17 \\ x_1 & - & 4x_2 & - & 12x_3 & = & 22 \end{array}$$

Example 3 - Solve the linear system.

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 6x_3 & = & 12 \\ 2x_1 & + & 4x_2 & + & 12x_3 & = & -17 \\ x_1 & - & 4x_2 & - & 12x_3 & = & 22 \end{array} \qquad \left(\begin{array}{cccc} 1 & -2 & -6 & 12 \\ 2 & 4 & 12 & -17 \\ 1 & -4 & -12 & 22 \end{array} \right)$$

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$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 6x_3 & = & 12 \\ 2x_1 & + & 4x_2 & + & 12x_3 & = & -17 \\ x_1 & - & 4x_2 & - & 12x_3 & = & 22 \end{array} \qquad \left(\begin{array}{cccc} 1 & -2 & -6 & 12 \\ 2 & 4 & 12 & -17 \\ 1 & -4 & -12 & 22 \end{array} \right)$$

$$-2 \cdot \text{Row1} + \text{Row2} \mapsto \text{Row2}$$

Example 3 - Solve the linear system.

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 6x_3 & = & 12 \\ 2x_1 & + & 4x_2 & + & 12x_3 & = & -17 \\ x_1 & - & 4x_2 & - & 12x_3 & = & 22 \end{array} \qquad \left(\begin{array}{cccc} 1 & -2 & -6 & 12 \\ 2 & 4 & 12 & -17 \\ 1 & -4 & -12 & 22 \end{array}\right)$$

$$-2 \cdot \text{Row1} + \text{Row2} \mapsto \text{Row2}$$

$$-\text{Row1} + \text{Row3} \mapsto \text{Row3}$$

Example 3 - Solve the linear system.

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 6x_3 & = & 12 \\ 2x_1 & + & 4x_2 & + & 12x_3 & = & -17 \\ x_1 & - & 4x_2 & - & 12x_3 & = & 22 \end{array} \qquad \left(\begin{array}{cccc} 1 & -2 & -6 & 12 \\ 2 & 4 & 12 & -17 \\ 1 & -4 & -12 & 22 \end{array} \right)$$

$$-2 \cdot \text{Row1} + \text{Row2} \mapsto \text{Row2}$$
$$-\text{Row1} + \text{Row3} \mapsto \text{Row3}$$

$$\left(\begin{array}{cccc} 1 & -2 & -6 & 12 \\ 0 & 8 & 24 & -41 \\ 0 & -2 & -6 & 10 \end{array} \right)$$

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$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 6x_3 & = & 12 \\ 2x_1 & + & 4x_2 & + & 12x_3 & = & -17 \\ x_1 & - & 4x_2 & - & 12x_3 & = & 22 \end{array} \qquad \left(\begin{array}{cccc} 1 & -2 & -6 & 12 \\ 2 & 4 & 12 & -17 \\ 1 & -4 & -12 & 22 \end{array} \right)$$

$$-2 \cdot \text{Row1} + \text{Row2} \mapsto \text{Row2}$$
$$-\text{Row1} + \text{Row3} \mapsto \text{Row3}$$

$$\left(\begin{array}{cccc} 1 & -2 & -6 & 12 \\ 0 & 8 & 24 & -41 \\ 0 & -2 & -6 & 10 \end{array} \right)$$

$$\text{Row2} \longleftrightarrow \text{Row3}$$

$$\begin{pmatrix} 1 & -2 & -6 & 12 \\ 0 & -2 & -6 & 10 \\ 0 & 8 & 24 & -41 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & -6 & 12 \\ 0 & -2 & -6 & 10 \\ 0 & 8 & 24 & -41 \end{pmatrix}$$

$$4 \cdot \text{Row2} + \text{Row3} \mapsto \text{Row3}$$

$$\begin{pmatrix} 1 & -2 & -6 & 12 \\ 0 & -2 & -6 & 10 \\ 0 & 8 & 24 & -41 \end{pmatrix}$$

$$4 \cdot \text{Row2} + \text{Row3} \mapsto \text{Row3}$$

$$\begin{pmatrix} 1 & -2 & -6 & 12 \\ 0 & -2 & -6 & 10 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & -6 & 12 \\ 0 & -2 & -6 & 10 \\ 0 & 8 & 24 & -41 \end{pmatrix}$$

$$4 \cdot \text{Row2} + \text{Row3} \mapsto \text{Row3}$$

$$\begin{pmatrix} 1 & -2 & -6 & 12 \\ 0 & -2 & -6 & 10 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 6x_3 & = & 12 \\ & - & 2x_2 & - & 6x_3 & = & 10 \\ & & & & 0 & = & -1 \end{array}$$

$$\begin{pmatrix} 1 & -2 & -6 & 12 \\ 0 & -2 & -6 & 10 \\ 0 & 8 & 24 & -41 \end{pmatrix}$$

$$4 \cdot \text{Row2} + \text{Row3} \mapsto \text{Row3}$$

$$\begin{pmatrix} 1 & -2 & -6 & 12 \\ 0 & -2 & -6 & 10 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \begin{array}{rcrcrcr} x_1 & - & 2x_2 & - & 6x_3 & = & 12 \\ & - & 2x_2 & - & 6x_3 & = & 10 \\ & & & & 0 & = & -1 \end{array}$$

Since the equation 0 = -1 is *never* true, this system has no solutions. That is,

the system is inconsistent.