Introduction to Abstract Mathematics — Fall 2013

Assignment 10.1 — Due November 15

Exercise 1. Let A be a set and let $\pi \subseteq \mathcal{P}(A)$ have the following two properties:

- $\varnothing \notin \pi$;
- for all $a \in A$, there is a unique $P \in \pi$ so that $a \in P$.

Prove that if we define, for any $a, b \in A$, $a \sim b$ if and only if there is a $P \in \pi$ so that $a, b \in P$, then $\sim$ is an equivalence relation.

Exercise 2. Let $f : A \to B$ be a function and let $R = \{(x, y) \mid f(x) = f(y)\} \subseteq A^2$. Prove that R is an equivalence relation on A.

Exercise 3. Given sets A, B and C, and relations $R \subseteq A \times B$ and $S \subseteq B \times C$, their *composite* is the relation $S \circ R \subseteq A \times C$ defined by

$$(a, c) \in S \circ R \text{ if and only if there is a } b \in B \text{ so that } (a, b) \in R \text{ and } (b, c) \in S.$$

a. Prove that if R and S are functions, then so is $S \circ R$.

b. Prove that if $S \circ R$ and S are both functions, then for any $a \in A$ there is a $b \in B$ so that aRb. Does this mean R is necessarily a function as well?