INTRODUCTION TO ABSTRACT MATHEMATICS | ASSIGNMENT 11.1
FALL 2013 | DUE NOVEMBER 22

Exercise 1. Let $\mathcal{S}$ denote the collection of all sets. Given $A, B \in \mathcal{S}$, define $A \sim B$ if and only if there is a bijection $f : A \to B$. Prove that $\sim$ is an equivalence relation on $\mathcal{S}$.

Exercise 2. Define $f : \mathbb{N} \to \mathbb{Z}$ as follows:

$$f(n) = \begin{cases} -n/2, & \text{if } n \text{ is even;} \\ (n-1)/2, & \text{if } n \text{ is odd.} \end{cases}$$

Prove that f is a bijection, i.e. that $|\mathbb{N}| = |\mathbb{Z}|$.

Exercise 3. Let $f : A \to B$. Prove that if f is a surjection and B is infinite, then A is infinite, and $|B| \leq |A|$. [*Suggestion:* For the cardinality comparison, use exercise 10.2.3]