Exercise 1. Given $A \in \mathcal{P}(\mathbb{N})$, define a binary sequence $\{b_n(A)\}_{n=1}^{\infty}$ by

$$b_n(A) = \begin{cases} 1, & \text{if } n \in A, \\ 0, & \text{if } n \notin A. \end{cases}$$

If B denotes the set of all binary sequences, show that the function $f : \mathcal{P}(\mathbb{N}) \to B$ given by $A \mapsto \{b_n(A)\}_{n=1}^{\infty}$ is a bijection.

Exercise 2. Show that the function $f : (0,1) \to \mathbb{R}$ given by $f(x) = \dfrac{2x-1}{x(1-x)}$ is a bijection.