Introduction to Abstract Mathematics — Assignment 12.2
Fall 2013 — Due December 4

Exercise 1. Show that subtraction, as a binary operation on $\mathbb{Z}$, is neither associative nor commutative. Do the same for division, as an operation on $\mathbb{Q} - \{0\}$.

Exercise 2. For $a, b \in \mathbb{R}$, define $a * b = \frac{a+b}{2}$. Show that $*$ is a commutative, but non-associative binary operation on $\mathbb{R}$.

Exercise 3. Let A, B, C, D be sets and let $f : A \to B$, $g : B \to C$ and $h : C \to D$. be functions. Prove that $h \circ (g \circ f) = (h \circ g) \circ f$, i.e. that function composition is associative.

Exercise 4. Recall that the set of *symmetries of the square* is

$$D_4 = \{R_0, R_1, R_2, R_3, H, V, F_1, F_2\},$$

where R_k is counterclockwise rotation by $90k$ degrees, H is the flip across the horizontal line of symmetry, V is the flip across the vertical line of symmetry, F_1 is the flip across the slope 1 diagonal, and F_2 is the flip across the slope -1 diagonal. Construct the Cayley table for composition as a binary operation on D_4.