Introduction to Abstract Mathematics — Assignment 13.1
Fall 2013 — Due December 10

Exercise 1. Let $m \in \mathbb{N}$, $m \geq 2$. Show that the binary operations $+_m$ and $\times_m$ on $\mathbb{Z}/m\mathbb{Z} = \{[0], [1], \ldots, [m-1]\}$ are well-defined. That is, given $[a], [a'], [b], [b'] \in \mathbb{Z}/m\mathbb{Z}$ with $[a] = [a']$ and $[b] = [b']$, prove the following.

a. $[a + b] = [a' + b']$.

b. $[ab] = [a'b']$.

Exercise 2. Construct the Cayley tables for $\mathbb{Z}/6\mathbb{Z}$ under the binary operations $+_6$ and $\times_6$.

Exercise 3. Let G be a group and let $a, b \in G$. Prove that $(ab)^{-1} = b^{-1}a^{-1}$. Generalize.

Exercise 4. Let G be a group. Given $h \in G$, define $\rho_h : G \to G$ by $\rho(g) = gh$.

a. Prove that ρ_h is a bijection.

b. Prove that the function $R : G \to S(G)$ given by $R(h) = \rho_h$ is an injection, and that for any $g, h \in G$, $R(gh) = R(h)R(g)$.