Exercise 1. Let P and Q be statements. Verify the following logical equivalences by either constructing a truth table or by using established equivalences.

  a. $P \wedge P \cong P \vee P \cong P$

  b. $P \rightarrow Q \cong \neg Q \rightarrow \neg P$

  c. $(P \rightarrow Q) \wedge (\neg P \rightarrow Q) \cong Q$

  d. $(P \wedge \neg Q) \rightarrow \neg P \cong P \rightarrow Q$

Exercise 2. If P, Q and R are statements, are $P \rightarrow (Q \vee R)$ and $(P \rightarrow Q) \vee (P \rightarrow R)$ logically equivalent? Does one imply the other?

Exercise 3. Let A and B be symbolic statements built from the statement variables and logical connectives. Suppose that A is a contradiction. Show that A implies B. Conversely, show that if B implies A, then B is also a contradiction.

Exercise 4. Let H, C and X be symbolic statements built from statements variables and logical connectives. Suppose that X is a contradiction. Show that if $H \wedge (\neg C)$ implies X, then H implies C.

Exercise 5. Let A, B and C be symbolic statements built from statement variables and logical connectives. Show that if A implies B, then $B \rightarrow C$ implies $A \rightarrow C$.