INTRODUCTION TO ABSTRACT MATHEMATICS — ASSIGNMENT 2.2
FALL 2013 — DUE SEPTEMBER 13

Exercise 1. Use a proof by contradiction to show that if $n = 2k + 1$ for some integer k, then n is odd. [*Note:* While intuitively obvious, this fact does require some kind of proof, since *odd* (the negation of *even*) simply means "not divisible by 2."] Conclude that n is odd if and only if $n = 2k + 1$ for some integer k.

Exercise 2. Use the preceding exercise to prove that an integer n is even if and only if n^2 is even.

Exercise 3. Consider the following result.

Lemma. *If n is an integer, then there are (unique) integers k and r, with $0 \leq r < 3$, so that $n = 3k + r$.*

Assuming the truth of the lemma, prove that for any integer n, the quantity

$$\frac{n(n+1)(2n+1)}{6}$$

is an integer.

Exercise 4. Let P, Q and R be statements.

- **a.** Show that $(P \to Q) \wedge (R \to \neg Q) \not\cong P \to \neg R$.
- **b.** Nonetheless, explain why a proof of $P \to Q$ and $R \to \neg Q$ would be enough to establish $P \to \neg R$.