Introduction to Abstract Mathematics
Fall 2013

Assignment 3.1
Due September 20

Exercise 1. Express the following statements symbolically, and determine if they are true or false. You may assume that the universe of discourse is $\mathbb{R}$.

a. For all $x \geq -1/4$, there is a y so that $y(y+1) = x$.

b. There is a y so that for all $x \geq -1/4$, $y(y+1) = x$.

Exercise 2. The technical definition of the statement $\lim_{x \to 0^+} \frac{1}{x} = \infty$ is the following: for any $M > 0$ there is an $\epsilon > 0$ so that $\frac{1}{x} > M$ whenever $0 < x < \epsilon$.

a. Express this statement symbolically.

b. Negate the symbolic expression in part **a**, and write a (meaningful!) equivalent statement in English.

c. Prove or disprove the original statement.

Exercise 3. Recall that we defined $\exists! x(P(x))$ to have the same meaning as

$$(\exists x(P(x))) \wedge (\forall x_1 \forall x_2 (P(x_1) \wedge P(x_2) \to x_1 = x_2)).$$

Negate this statement symbolically, and express the negation in English.

Exercise 4. Consider the following statements:

$A =$ "You can fool all of the people some of the time."
$B =$ "You can fool some of the people all of the time."
$C =$ "You can't fool all of the people all of the time."

If $F(x,t) =$ "Person x is fooled at time t," express each of these statements symbolically.