Introduction to Abstract Mathematics — Assignment 3.2

Fall 2013 — Due September 20

**Exercise 1.** Conjecture and prove a formula for the sum of the first $n$ Fibonacci numbers.

**Exercise 2.** For $n \geq 1$, the $n$th *harmonic number* is defined to be

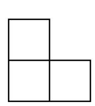
$$H_n = \sum_{k=1}^{n} \frac{1}{k}.$$

Prove that for all $n \geq 0$, $H_{2^n} \geq 1 + \frac{n}{2}$.

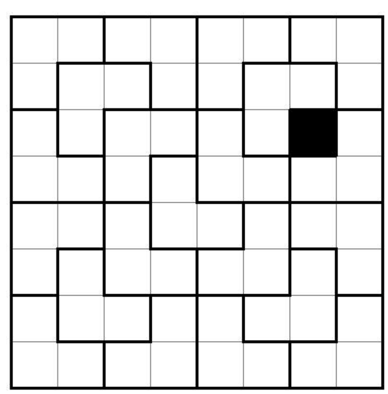
**Exercise 3.** What is wrong with the following "proof" that all horses have the same color?

> We induct on $n$, the number of horses. If $n = 1$, then since every horse has the same color as itself, the result is true. Now suppose that, for some $n$, we have proven that every horse in a group of $n$ horses has the same color. Consider a group of $n + 1$ horses. By the inductive hypothesis, the first $n$ horses in this group all have the same color, and the last $n$ also all have the same color. Since the $n$th horse is in both groups, all $n+1$ horses must have its color, and hence the whole group has a single color. By induction, this completes the proof.

**Exercise 4.** Show that for any $n \geq 1$, a $2^n \times 2^n$ grid with one square removed can be completely covered with (non-overlapping) L-shaped tiles (shown below). An example in the case $n = 3$ is given below.



L-shaped tile



Sample tiling