Introduction to Abstract Mathematics
Fall 2013

Assignment 4.1
Due September 27

Exercise 1. Let F_n denote the nth Fibonacci number (where, as in class, we set $F_0 = 0$ and $F_1 = 1$). Prove that for all $n \geq 0$,

$$F_n = \frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right).$$

Exercise 2.[The Division Algorithm] Let $m > 0$ be an integer. Prove that for every integer $n \geq 1$, there exist integers q and r, with $0 \leq r < m$, so that $n = qm + r$.

Exercise 3. Given a sequence of non-negative real numbers

$$a_1 \geq a_2 \geq \cdots \geq a_{2n+1} \geq 0,$$

prove that

$$a_1^2 - a_2^2 + a_3^2 - \cdots + a_{2n+1}^2 \geq (a_1 - a_2 + a_3 - \cdots + a_{2n+1})^2.$$