Introduction to Abstract Mathematics — Assignment 4.2
Fall 2013 — Due September 27

Exercise 1. Given integers n and k, with $0 \leq k \leq n$, the (n, k) *binomial coefficient* is

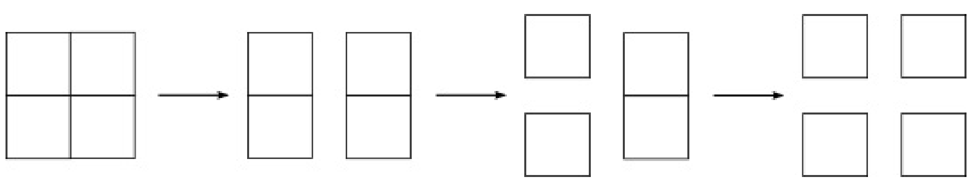
$$\binom{n}{k} = \frac{n!}{k!(n-k)!},$$

where we define $0! = 1$.

a. Prove that if $1 \leq k \leq n-1$, then $\binom{n-1}{k-1} + \binom{n-1}{k} = \binom{n}{k}$.

b. Prove that every binomial coefficient is an integer.

Exercise 2. Suppose you are given a rectangular chocolate bar made up of $m \times n$ squares of chocolate ($m, n \geq 1$). Your task is to divide it into mn individual squares by breaking it (or any of the intermediate pieces) along any of its horizontal or vertical perforations. An example in the 2×2 case is shown below.



Prove that no matter how you choose to break the chocolate bar, it will always require exactly $mn - 1$ breaks.

Exercise 3. Let F_n denote the nth Fibonacci number. Prove that for all $m, n \geq 1$, if m divides n, then F_m divides F_n. [*Suggestion:* Instead prove that F_m divides F_{km} for all $k, m \geq 1$. Why is this equivalent?]