Introduction to Abstract Mathematics
Assignment 5.3 FALL 2013

Exercise 1. For each pair $(a, b)$, find $\operatorname{gcd}(a, b)$ as well as $x$ and $y$ so that $x a+y b=\operatorname{gcd}(a, b)$.
a. $(14,23)$
b. $(130,150)$
c. $(34,144)$

Exercise 2. Let $a, b, c \in \mathbb{Z}$. Prove that if $a|c, b| c$ and $\operatorname{gcd}(a, b)=1$, then $a b \mid c$.

Exercise 3. Recall the following statement from class:
Let $p$ be a prime, $a, b \in \mathbb{Z}$. If $p \mid a b$, then $p \mid a$ or $p \mid b$.
a. Prove that if $p \nmid b$, then $\operatorname{gcd}(p, b)=1$. [Suggestion: What are the possible values for the gcd of any number with a prime?]
b. We showed in class that the statement above is equivalent to "If $p \nmid b$, then $p \mid a b$ implies $p \mid a$." Use part a to deduce this as a corollary of another fact we proved in class.

