Introduction to Abstract Mathematics — Assignment 7

Fall 2013 — Due October 16

Exercise 1. Let X be a set and $\mathcal{C} \subseteq \mathcal{P}(X)$. Prove DeMorgan's Law in the form

$$\left(\bigcup_{A\in\mathcal{C}} A\right)^c = \bigcap_{A\in\mathcal{C}} (A^c)$$

by using set complements and the "opposite" law (i.e. that the complement of the intersection is the union of the complements).

Exercise 2. For $n \in \mathbb{N}$, define $I_n = (1 - 1/n, 1 + 1/n)$, an open interval in $\mathbb{R}$. Find explicit descriptions (with proof!) of the sets

$$\bigcup_{n\in\mathbb{N}} I_n \text{ and } \bigcap_{n\in\mathbb{N}} I_n.$$

You may feel free to use the following consequence of the Archimedean Law: given any positive $\epsilon \in \mathbb{R}$, there is an $n \in \mathbb{N}$ so that $1/n < \epsilon$.

Exercise 3. Let A, B, C, D be sets. Prove that if $A \subseteq C$ and $B \subseteq D$, then $A \times B \subseteq C \times D$. Is it true that *every* subset of $C \times D$ has the form $A \times B$ for some $A \subseteq C$ and $B \subseteq D$?

Exercise 4. Let A, B, C, D be sets. Prove that

$$A \times (B - D) \cup (A - C) \times B \subseteq A \times B - C \times D.$$

Based on our work in class, this containment completes the proof that these two sets are actually equal.