Exercise 1. Prove that the following are partial orderings on the sets indicated. Which are total orderings?

a. The relation $D = \{(a, b) \mid a|b\}$ on $\mathbb{N}$.

b. The relation $A = \{((x, y), (z, w)) \mid x < z, \text{ or } x = z \text{ and } y \leq w\}$ on $\mathbb{R}^2$.

c. The relation $C = \{(A, B) \mid A \subseteq B\}$ on $\mathcal{P}(X)$, where X is any set. [*Remark:* Your answer to the question of totality will depend on the number of elements in X.]

Exercise 2. Let R_1 and R_2 be relations on a set A, i.e. $R_1, R_2 \subseteq A \times A$. Which of the "regularity" properties (reflexivity, symmetry, transitivity, antisymmetry, totality) are retained by $R = R_1 \cap R_2$? That is, if R_1 and R_2 both possess one of these properties (e.g. both are reflexive), is the same true of R? Answer the same questions with $\cap$ replaced by $\cup$.

Exercise 3. Let $A = \{a, b, c\}$, $R = \{(a, a), (a, b), (b, c), (c, b)\}$.

a. Find the smallest reflexive relation S on A that contains R.

b. Find the smallest transitive relation S on A that contains R.

c. Find the smallest symmetric relation S on A that contains R.