Introduction to Abstract Mathematics — Assignment 9.1
Fall 2013 — Due November 8

Exercise 1. Let $m \in \mathbb{N}$. Show that, under congruence modulo m on $\mathbb{Z}$, the equivalence classes $[0], [1], [2], \ldots, [m-1]$ are all distinct, and that every $n \in \mathbb{Z}$ belongs to one of them.

Exercise 2. Let $a, b \in \mathbb{Z}$ and set $d = \gcd(a, b)$. Write $a = a'd$ and $b = b'd$ for some $a', b' \in \mathbb{Z}$. Prove that if $\gcd(a, b) \neq 0$, then $\gcd(a', b') = 1$. [*Suggestion:* Use the "gcd as linear combination" theorem.]

Exercise 3. Let $\sim$ be the equivalence relation on $\mathbb{Z} \times \mathbb{N}$ defined by: $(p, q) \sim (r, s)$ if and only if $ps - qr = 0$.

a. Show that for any $k \in \mathbb{N}$ and $(p, q) \in \mathbb{Z} \times \mathbb{N}$, $(p, q) \sim (kp, kq)$.

b. Show that if $(p, q), (r, s) \in \mathbb{Z} \times \mathbb{N}$, $(p, q) \sim (r, s)$, and $\gcd(p, q) = \gcd(r, s) = 1$, then $(p, q) = (r, s)$.

c. Show that each equivalence class $[(p, q)] \in (\mathbb{Z} \times \mathbb{N})/\!\sim$ contains *exactly one* element (s, t) with $\gcd(s, t) = 1$.