Exercise 1. Given $x \in \mathbb{R}$, we define $\lfloor x \rfloor$ to be the greatest integer less than or equal to x, i.e. $\lfloor x \rfloor = n \in \mathbb{Z}$ if and only if $n \leq x < n+1$. $\lfloor x \rfloor$ is sometimes called the *floor function*. Let $\sim$ be the equivalence relation of Exercise 8.3.1.

a. Show that for any $x \in \mathbb{R}$, $x \sim (x - \lfloor x \rfloor)$.

b. Use part **a** to show that $\mathbb{R}/{\sim} = \{[y] \mid y \in [0,1)\}$, and that these classes are all distinct. Since $[0] = [1]$, this shows that (in a sense we will not make more precise) $\mathbb{R}/\sim$ is actually a "circle."

Exercise 2. Let $\sim$ be the equivalence relation on $\mathbb{R}^n$ of Exercise 8.3.3. We define *n-dimensional (real) projective space* to be $\mathbb{P}^n = \mathbb{R}^{n+1}/{\sim}$. Show that

$$\mathbb{P}^1 = \{[(x,1)] \mid x \in \mathbb{R}\} \cup \{[(1,0)]\},$$

and that each of these classes is distinct. This shows that (in a sense we will not make more precise) $\mathbb{P}^1$ is simply $\mathbb{R}$ together with an additional "point at infinity."