CALCULUS I
FALL 2015

MIDTERM 2 REVIEW

Exercise 1. Find y'.

a. $y = \ln|\sec 5x + \tan 5x|$

b. $\sin(xy) = x^2 - y$

c. $y = (\arcsin(2x))^2$

d. $y = \sin^2\left(\cos\sqrt{\sin \pi x}\right)$

e. $y = \dfrac{\sqrt{x+1}(2-x)^5}{(x+3)^7}$

f. $y = \ln(x \ln x)$

Exercise 2. Find y'' if $x^6 + y^6 = 1$.

Exercise 3. Find equations for the tangent lines to the ellipse $x^2 + 2y^2 = 1$ that have slope 1.

Exercise 4. Use a linear approximation to estimate $\tan 44°$.

Exercise 5. The circumference of a sphere was measured to be 84 cm with a possible error of 0.5 cm.

a. Use differentials to estimate the maximum error in the calculated surface area. What is the percentage error?

b. Use differentials to estimate the maximum error in the calculated volume. What is the percentage error?

Exercise 6. A plane flies horizontally at an altitude of 5km and passes directly over a tracking telescope on the ground. When the angle of elevation is $\pi/3$, this angle is decreasing at a rate of $\pi/6$ rad/min. How fast is the plane traveling at that time?

Exercise 7. A balloon is rising at a constant speed of 5 ft/s. A girl is cycling along a straight road at a speed of 15 ft/s. When she passes under the balloon, it is 45 ft above her. How fast is the distance between the girl and the balloon increasing 3 s later?

Exercise 8. Find the absolute maximum and minimum values of f on the given interval.

a. $f(x) = x\sqrt{4 - x^2}, \ [-1, 2]$

b. $f(t) = t^{2/3}(10 - t), \ [-2, 8]$

c. $f(\theta) = \theta - 2\arctan(\theta), \ [0, 4]$

d. $f(x) = xe^{-x^2/8}, \ [-1, 4]$

Exercise 9. Show that the equation $x^3 - 15x + 1 = 0$ has exactly one real solution in the interval $[-2, 2]$.

Exercise 10. Is there a differentiable function f such that $f(0) = 2$, $f(2) = -5$ and $f'(x) \geq -3$ for all x?

Exercise 11. Let $f(x) = x^4(x - 1)^3$.

a. Find the intervals on which f is increasing or decreasing.

b. Find the local extreme values of f.

c. Find the intervals on which the graph of f is concave up or concave down.

d. Find the x-coordinates of the inflection points of the graph of f.

Exercise 12. Evaluate the following limits.

a. $\lim_{x \to 0} \dfrac{1 - 8x^2 - \cos 4x}{x^4}$

b. $\lim_{x \to 0^+} x \left(\ln x\right)^2$

c. $\lim_{x \to 4} \left(\dfrac{1}{\sqrt{x} - 2} - \dfrac{4}{x - 4} \right)$

d. $\lim_{x \to 0^+} x^{\sin x}$