

Calculus I Fall 2015

MIDTERM 3 REVIEW

Exercise 1. Design a cylindrical can of volume 900 cm³ so that it uses the least amount of metal. In other words, minimize the surface area of the can (include its top and bottom).

Exercise 2. Use Newton's method to find all solutions of the equation $\sin x = x^2 - 3x + 1$ correct to six decimal places.

Exercise 3. A driver involved in an accident claims he was going less than 60 mi/h when he slammed on his brakes. When investigators tested his car, they found that the maximum deceleration its brakes could provide was 15 ft/s². Given that the skid marks left by the car at the scene of the accident were 210 ft long, is the driver telling the truth?

Exercise 4. Evaluate the following integrals.

$$\mathbf{a.} \quad \int \frac{1+\sqrt{x}+x}{x} \, dx$$

b.
$$\int_{-1}^{2} x - 2|x| \, dx$$

$$\mathbf{c.} \quad \int x(2x+5)^8 \, dx$$

d.
$$\int_{0}^{2} r \sqrt{5 - \sqrt{4 - r^2}} \, dr$$

e.
$$\int \frac{\tan(\ln x)}{x}$$

$$\mathbf{f.} \quad \int_{1}^{2} \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}} \, dx$$

Exercise 5. Evaluate the limit

$$\lim_{n\to\infty}\sum_{i=1}^n\frac{i\sqrt{1+i^2/n^2}}{n^2}$$

by first expressing it as a definite integral.

Exercise 6. If $f(x) = \int_0^x (4-t^2)e^{t^2} dt$, on what interval(s) is f increasing? On what interval(s) is f concave down?

Exercise 7. Find g'(x) if

$$g(x) = \int_{x}^{2x} \frac{u^2 - 1}{u^2 + 1} du.$$

Exercise 8. Find the area of the region enclosed by the curves $y = x^2 - 4$ and $y = x^3 - 4x$.

Exercise 9. Find the volume of the solid whose base is the region enclosed by the parabola $y = 1-x^2$ and the x-axis, and whose cross-sections perpendicular to the y-axis are semicircles.