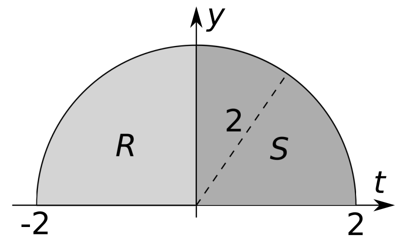
6. Let $G(x) = \int_0^x \sqrt{4 - t^2}\, dt$.

a. Compute $G(-2)$, $G(0)$ and $G(2)$. [*Suggestion:* Draw a graph and think geometrically.]

The graph of $y = \sqrt{4 - t^2}$ is a semicircle of radius 2:



$$G(-2) = \int_0^{-2} \sqrt{4 - t^2}\, dt = -\int_{-2}^{0} \sqrt{4 - t^2}\, dt = -(\text{area of } R) = -\frac{1}{4}\pi 2^2 = \boxed{-\pi}$$

$$G(0) = \int_0^0 \sqrt{4 - t^2}\, dt = \boxed{0}$$

$$G(2) = \int_0^2 \sqrt{4 - t^2}\, dt = (\text{area of } S) = \frac{1}{4}\pi 2^2 = \boxed{\pi}$$

b. Compute $G'(x)$.

According to the Fundamental Theorem of Calculus

$$G'(x) = \frac{d}{dx} \int_0^x \sqrt{4 - t^2}\, dt = \boxed{\sqrt{4 - x^2}}.$$

c. If $H(x) = \int_0^{x^3} \sqrt{4 - t^2}\, dt$, compute $H'(x)$.

Since $H(x) = G(x^3)$, we use the chain rule and part **b**:

$$H'(x) = G'(x^3) \cdot 3x^2 = \sqrt{4 - (x^3)^2} \cdot 3x^2 = \boxed{3x^2\sqrt{4 - x^6}}$$

7. Express $\int_{\pi/2}^{\pi} x \sin x \, dx$ as a limit of Riemann sums. *Do not evaluate this limit.*

If we partition $[\pi/2, \pi]$ into n equally sized subintervals, then each will have length

$$\Delta x = \frac{\pi - (\pi/2)}{n} = \frac{\pi}{2n}.$$

If we use right-handed Riemann sums, then our selection points are

$$x_i^* = \frac{\pi}{2} + i\Delta x = \frac{\pi}{2} + \frac{i\pi}{2n}.$$

Hence

$$\int_{\pi/2}^{\pi} x \sin x \, dx = \lim_{n\to\infty} \sum_{i=1}^{n} x_i^* \sin(x_i^*) \Delta x = \boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{\pi}{2} + \frac{i\pi}{2n}\right) \sin\left(\frac{\pi}{2} + \frac{i\pi}{2n}\right) \frac{\pi}{2n}}$$