Math 1311-2 Fall 2006

Calculus I

First Midterm Exam

Friday, September 22, 10:30 AM - 11:20 AM

Your name (please print):

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted.** You must justify all of your answers to receive credit.

You have **50 minutes** to work on all **8** problems. Please do all of your work on the paper provided.

The Honor Code requires that you neither give nor receive any aid on this exam.

FERPA waiver: By my signature I relinquish my FERPA rights in the following context: This exam paper may be returned en masse with others in the class and I acknowledge that I understand my score may be visible to others.

If I choose not to relinquish my FERPA rights, I understand that I will have to present my student ID at my instructor's office to retrieve my examination paper.

FERPA waiver signature: ______________________________

Problem	1	2	3	4	5	6	7	8	Total
Points	10	16	15	10	15	10	8	16	100
Score									

Problem 1. [10 points] If $f(x) = x^3 - x^2 + x$, use the Intermediate Value Property to show that there is a number c so that $f(c) = 10$. Be sure to explain why you may apply the Intermediate Value Property.

Problem 2. [16 points] Find equations for the two lines tangent to the curve

$$y = \frac{x-1}{x+1}$$

that are parallel to the line $x - 2y = 2$. [*Hint:* Parallel lines have the same slope.]

Problem 3. Evaluate the following limits, or explain why they do not exist. Be sure to justify your answers.

(a) [5 points]

$$\lim_{x \to 4} \frac{\sqrt{x^2+9}-5}{x-4}$$

(b) [5 points]

$$\lim_{x \to a} 7x^3 - 5x + \frac{\cos x}{x^2+1}$$

(c) [5 points]

$$\lim_{x \to \pi^+} \frac{x}{\sin x}$$

Problem 4. [10 points] A table of values for f, g, f' and g' is given.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

(a) If $h(x) = f(g(x))$, find $h'(1)$.

(b) If $H(x) = g(f(x))$, find $H'(1)$.

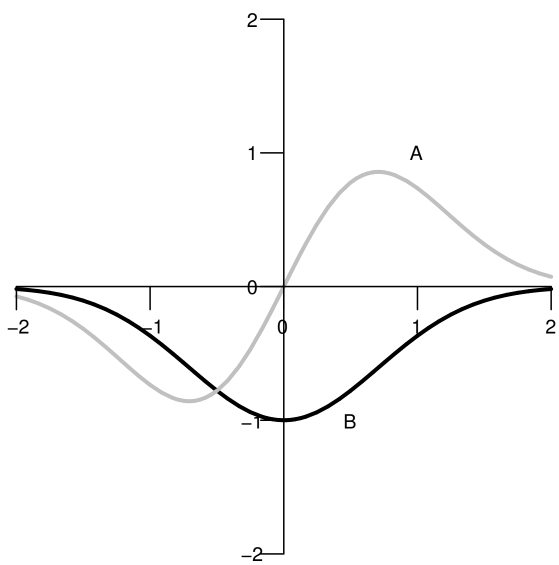
Problem 5. [15 points] Let

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

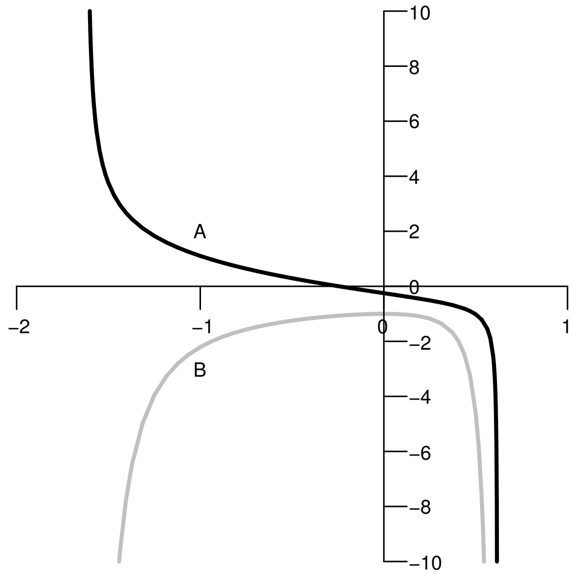
Use the definition of the derivative and the limit laws to show that $f'(0) = 0$.

Problem 6. [10 points] A particle moves along the curve $y = \sqrt{1 + x^3}$. As it reaches the point $(2, 3)$, the y-coordinate is increasing at a rate of 4 cm/s. How fast is the x-coordinate of the point changing at that instant?

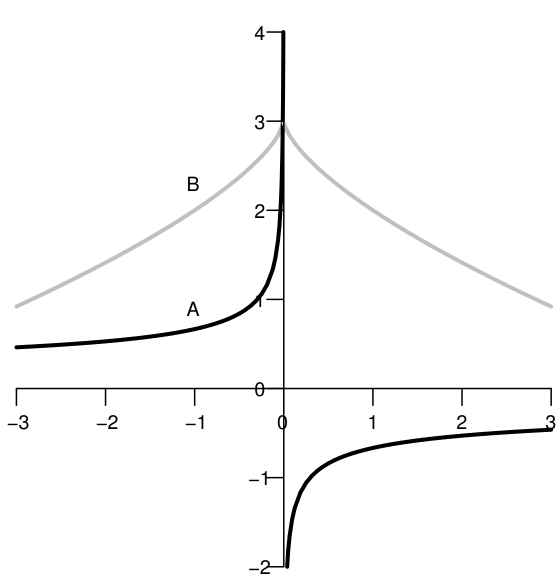
Problem 7. [2 points each] In each figure below the graph of a function $f(x)$ is shown together with the graph of its derivative $f'(x)$. In each case, identify which curve is the graph of $f(x)$ and which is the graph of $f'(x)$. You do not need to justify your answers and no partial credit is possible.



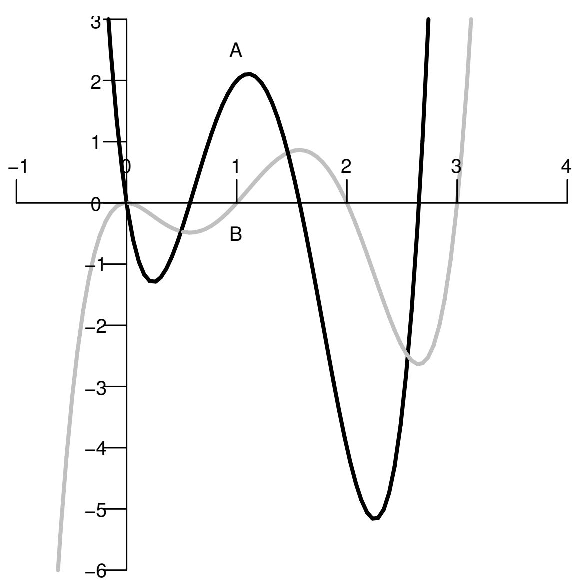
(a) $f =$ ____________, $f' =$ ____________



(b) $f =$ ____________, $f' =$ ____________



(c) $f =$ ____________, $f' =$ ____________



(d) $f =$ ____________, $f' =$ ____________

Problem 8. Recall that for a real number x, $[\![x]\!]$ denotes the greatest integer less than or equal to x. Let

$$v(x) = [\![x]\!] + [\![-x]\!].$$

(a) [10 points] Compute

$$\lim_{x\to 2} v(x).$$

[*Hint:* Compute the the two one-sided limits and show that they agree.]

(b) [3 points] Compute $v(2)$.

(c) [3 points] Use (a) and (b) to show that $v(x)$ is discontinuous at $x = 2$.

(Work Page)