Math 1311 Fall 2007

Calculus I

Second Midterm Exam

Tuesday, October 23, 7:00 pm - 9:00 pm

Your name (please print):

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted.** You must justify all of your answers to receive credit. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided.

The Honor Code requires that you neither give nor receive any aid on this exam.

If you are bound by the Academic Honor Code, please indicate that you have read and understood these guidelines by signing your name in the space provided:

Pledged: _____

Do not write below this line

Problem	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Points	10	20	10	20	15	20	5
Score							

Total:_____

1.

(a) Find the linear approximation $L(x)$ to the function

$$f(x) = (1+x)^{100}$$

near the point $a = 0$.

(b) Use part (a) to estimate 1.001^{100}.

2. Find and classify the critical points of the following functions.

(a) $q(x) = (2x+1)e^{-x^2}$

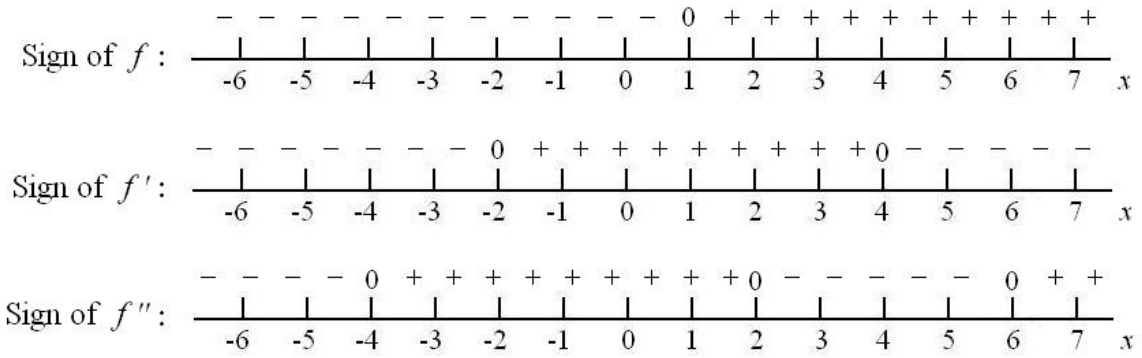
(b) $r(x) = x^2 + \dfrac{1}{x^2}$

3. Find the inflection points of the graph of the function

$$h(x) = 3x^{1/3} + 9x^{4/3}.$$

4. A box with a square base and an open top is to have a volume of 2 ft^3. The material for the bottom of the box costs 5 dollars per square foot while the material for the sides costs 1 dollar per square foot. What dimensions will minimize the total cost of this box? You *must* define all variables you introduce and indicate their units.

5. Suppose we are given the following information about a twice differentiable function $f(x)$:



$$f(4) = 6, \quad f(-4) = -4, \quad f(-2) = -6, \quad f(2) = 3, \quad f(6) = 4,$$

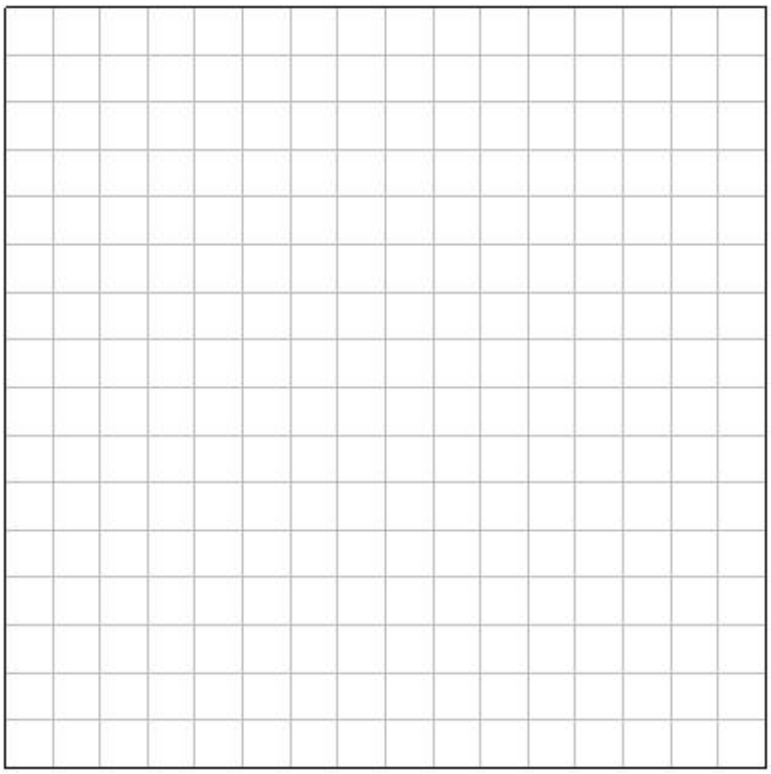
and

$$\lim_{x \to \infty} f(x) = 2, \quad \lim_{x \to -\infty} f(x) = -1.$$

(a) On which intervals is $f(x)$ increasing? On which intervals is $f(x)$ decreasing?

(b) On which intervals is $f(x)$ concave up? On which intervals is $f(x)$ concave down?

(c) *Carefully* sketch the graph of $y = f(x)$, being sure to label your axes as well as all local maxima and minima, inflection points and asymptotes (if any) of $f(x)$.



6. Evaluate the following limits.

(a) $\lim_{x \to 0} \dfrac{x - \sin x}{x^3}$

(b) $\lim_{x \to 1} x^{\frac{1}{1-x}}$

7. Suppose that f is continuous on $[0, 4]$ and differentiable on $(0, 4)$. If $f(0) = 1$ and $2 \leq f'(x) \leq 5$ for all x in $(0, 4)$ show that $9 \leq f(4) \leq 21$. [*Hint:* Apply the Mean Value Theorem on the interval $[0, 4]$.]