Math 1311-2 Fall 2006

Calculus I

Second Midterm Exam

Wednesday, October 18, 10:30 AM - 11:20 AM

Your name (please print):

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted.** You must justify all of your answers to receive credit.

You have **50 minutes** to work on all **8** problems. Please do all of your work on the paper provided.

The Honor Code requires that you neither give nor receive any aid on this exam.

FERPA waiver: By my signature I relinquish my FERPA rights in the following context: This exam paper may be returned en masse with others in the class and I acknowledge that I understand my score may be visible to others.

If I choose not to relinquish my FERPA rights, I understand that I will have to present my student ID at my instructor's office to retrieve my examination paper.

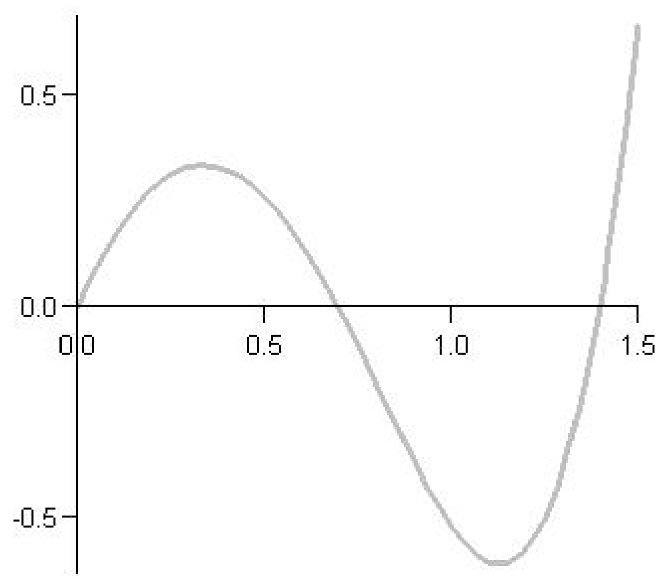
FERPA waiver signature: ____

Problem	1	2	3	4	5	6	7	8	Total
Points	10	20	10	10	15	10	15	10	100
Score									

Problem 1. The equation $3\sin(x^2) - 2x = 0$ has exactly one solution in the interval $[1/2, 1]$.

(a) [5 points] If x_n is the nth Newton's method approximation to the solution of the equation $3\sin(x^2) - 2x = 0$, write down an equation that expresses x_{n+1} in terms of x_n.

(b) [5 points] The function $f(x) = 3\sin(x^2) - 2x$ is plotted below. If $x_0 = 0.5$, carefully sketch the location of x_1.



Problem 2. Compute the derivatives of the following functions.

(a) [5 points]

$$f(x) = e^{\frac{-1}{x+1}}$$

(b) [5 points]

$$r(t) = \cot(e^{t^3+5t-7})$$

(c) [5 points]

$$h(x) = x^{\cos x^2}$$

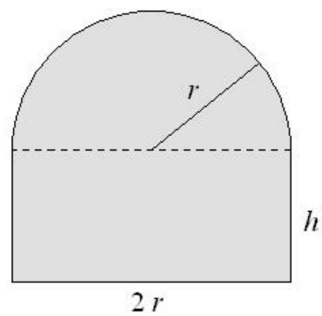
(d) [5 points]

$$g(\theta) = \ln\left(\frac{\sin\theta}{\theta}\right)$$

Problem 3. [10 points] Find the absolute maximum and absolute minimum values of $f(x) = 2x^3 - 9x^2 - 24x + 20$ on the interval $[0, 5]$.

Problem 4. [10 points] Use the linear approximation to estimate the amount of paint needed to apply a coat of paint 0.05 cm thick to a hemispherical dome with diameter 50 m. [*Hint:* The volume V of a sphere with radius r is given by $V = (4/3)\pi r^3$.]

Problem 5. Consider a semicircle of radius r that sits atop a rectangle of width $2r$ and height h, as shown in the figure below. The perimeter of the entire figure is fixed at 30 ft.



(a) [5 points] Express the area of the figure as a function of r alone.

(b) [5 points] Determine the interval in which your expression in part (a) is valid.

(c) [5 points] Find the dimensions of the figure that maximize its area. You may assume that the maximum does not occur at either endpoint of the interval you found in (b).

Problem 6. [10 points] Suppose that f is continuous on $[0, 4]$ and differentiable on $(0, 4)$. If $f(0) = 1$ and $2 \leq f'(x) \leq 5$ for all x in $(0, 4)$ show that $9 \leq f(4) \leq 21$. [*Hint:* Use the Mean Value Theorem.]

Problem 7. [15 points] Professor Daileda is fishing on the shore of a lake. A fish bites the hook at the end of the Professor's line and then proceeds to swim directly away from the shore at a speed of 2 ft/s. If the tip of the Professor's fishing pole is 7 ft above the surface of the water and the fish is at a depth of 6 ft, how quickly is fishing line coming off of the reel when the fish is 20 ft from the shore?

Problem 8. Consider the *astroid* defined by the equation $x^{2/3} + y^{2/3} = 20$.

(a) [4 points] Find dy/dx.

(b) [6 points] Find all points on the astroid where the tangent line has slope 2.

(Work Page)