

# MATH 1311 FALL 2015

## CALCULUS I

### THIRD MIDTERM EXAM

FRIDAY, DECEMBER 4

YOUR NAME (PLEASE PRINT):

**Instructions:** This is a closed book, closed notes exam. **Use of calculators or other electronic devices such as cell phones, mp3 players, etc. is not permitted.** Unless indicated otherwise, you must justify all of your answers to receive credit. Unjustified answers and/or disorganized or otherwise illegible work will receive partial credit at best. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided.

**The Honor Code requires that you neither give nor receive any aid on this exam.**

Please indicate that you have read and understood these guidelines by signing your name in the space provided:

Pledged: \_\_\_\_\_

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Do not write below this line

Problem	1	2	3	4	5	6	7	8
Points	15	12	8	5	6	8	8	8
Score								

**Total:**\_\_\_\_\_

1. Evaluate the following integrals. Simplify your answers as much as possible.

a.  $\int \sqrt[3]{x} (1 - x)^2 dx$

b.  $\int \sin(10t - 7) dt$

c.  $\int_0^1 \frac{(\arctan x)^4}{1 + x^2} dx$

**2.** A landscape architect wishes to enclose a rectangular garden of area  $900 \text{ m}^2$  on one side by a brick wall costing  $\$90/\text{m}$  and on the other three sides by a metal fence costing  $\$30/\text{m}$ .

**a.** Express the cost of the fence as a function of the length of one of its sides.

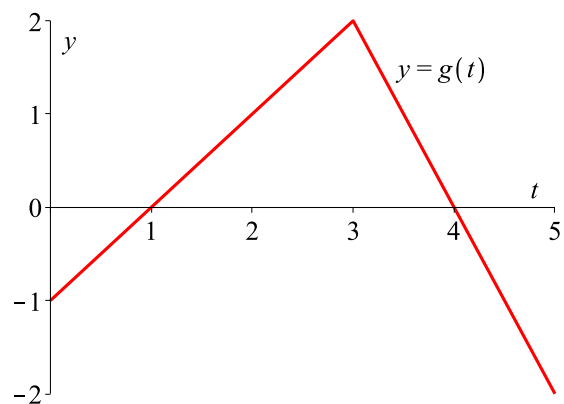
**b.** What is the domain of the cost function you found in part **a**?

**c.** Find the dimensions that minimize the cost of the fence. You must explain how you know that you have found the absolute minimum cost.

**3.** A particle is moving along the  $x$ -axis with acceleration at time  $t$  given by  $a(t) = 4 + 6t + 24t^2$ . Initially the particle is 3 units to the right of the origin. One second later it is 10 units to the right of the origin. Determine the position of the particle at any time  $t$ .

**4.** Express the integral  $\int_1^3 xe^x dx$  as a limit of Riemann sums. *Do not evaluate the limit.*

5. Consider the function  $g$  whose graph is shown below.



a. Evaluate  $\int_0^3 g(t) dt$ .

b. Evaluate  $\int_3^5 g(t) dt$ .

c. Evaluate  $\int_1^0 g(t) dt$ .

d. Determine  $a$  so that  $\int_0^a g(t) dt$  is as large as possible.

e. Determine  $b$  and  $c$  so that  $\int_b^c g(t) dt$  is as large as possible.

6. Let  $f(x) = x^2 + x - \cos x$ .

a. If Newton's method is used to find the critical numbers of  $f$ , what is the recursion relation needed?

b. Starting with the initial approximation  $x_1 = 0$ , use your answer to part a to compute  $x_2$  and  $x_3$ . Simplify your expressions as much as possible.

**7.** Compute the area of the region between the curves  $y = x^2 - 2x$  and  $y = 4 - x^2$ . Simplify your answer as much as possible.

8. The region in the first quadrant bounded by the lines  $y = x - 1$ ,  $y = 0$  and  $x = 2$  is rotated about the line  $y = -1$ . Find the volume of the resulting solid. Simplify your answer as much as possible.