Math 1311 Spring 2008

Calculus I

Third Midterm Exam

Tuesday, April 15, 7:00 pm - 9:00 pm

Your name (please print):

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted.** You must justify all of your answers to receive credit. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided.

The Honor Code requires that you neither give nor receive any aid on this exam.

If you are bound by the Academic Honor Code, please indicate that you have read and understood these guidelines by signing your name in the space provided:

Pledged: _____

Do not write below this line

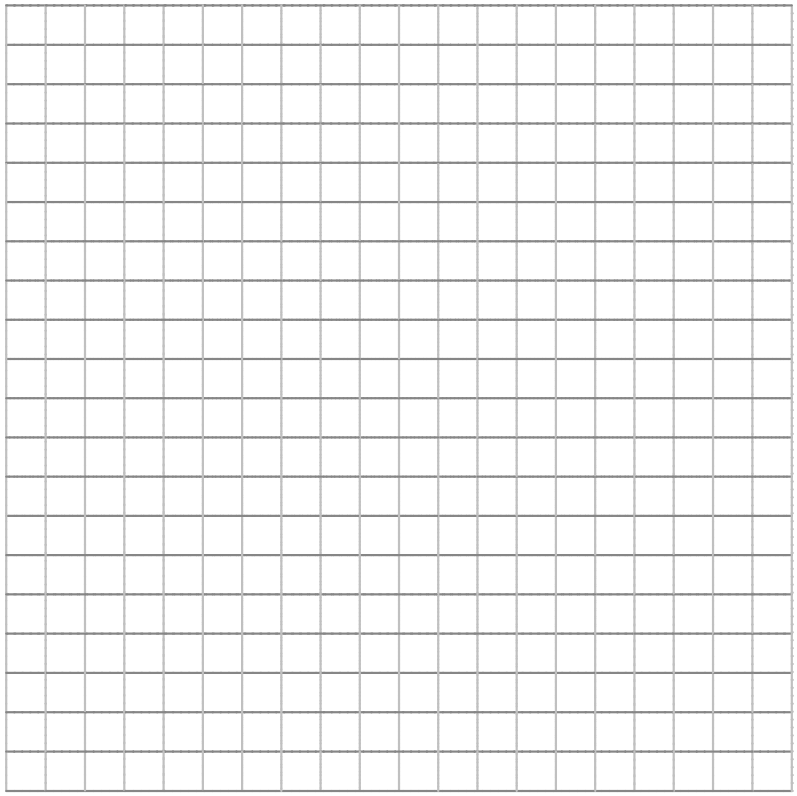
Problem	**1**	**2**	**3**	**4**	**5**	**6**
Points	20	20	20	18	12	10
Score						

Total:_____

1. Suppose you are told that

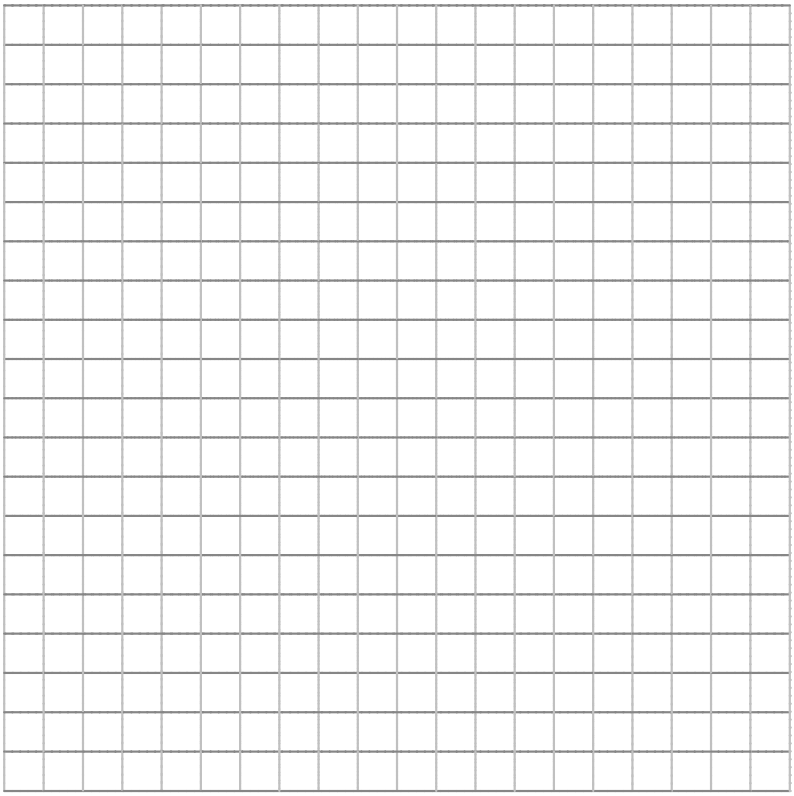
$$f(x) = \frac{1}{x(x-3)^2}, \qquad f'(x) = \frac{-3x+3}{x^2(x-3)^3}, \qquad f''(x) = \frac{12x^2-24x+18}{x^3(x-3)^4}.$$

Use this information to carefully sketch the graph of $y = f(x)$. Be sure to label the axes of your graph and indicate all asymptotes and critical points.



2. Sketch the graph of a function that satisfies the following conditions:

$h(0) = 0$, $h'(-2) = h'(1) = h'(9) = 0$,
$\lim_{x\to\infty} h(x) = 0$, $\lim_{x\to 6} h(x) = -\infty$,
$h'(x) < 0$ on $(-\infty, -2)$, $(1, 6)$, and $(9, \infty)$,
$h'(x) > 0$ on $(-2, 1)$ and $(6, 9)$,
$h''(x) > 0$ on $(-\infty, 0)$ and $(12, \infty)$,
$h''(x) < 0$ on $(0, 6)$ and $(6, 12)$.



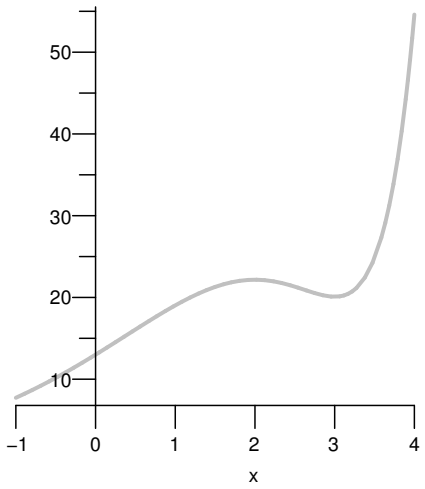
3. Match each of the following functions with its graph. No partial credit is possible.

$$q(x) = 4x^5 - 25x^4 + 40x^3 \quad \_\_\_\_\_$$

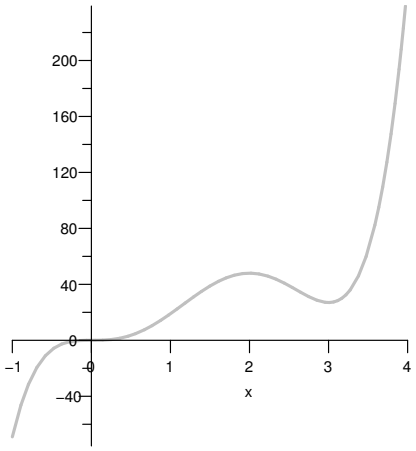
$$r(x) = 3x^4 - 20x^3 + 36x^2 \quad \_\_\_\_\_$$

$$s(x) = (x^2 - 7x + 13)e^x \quad \_\_\_\_\_$$

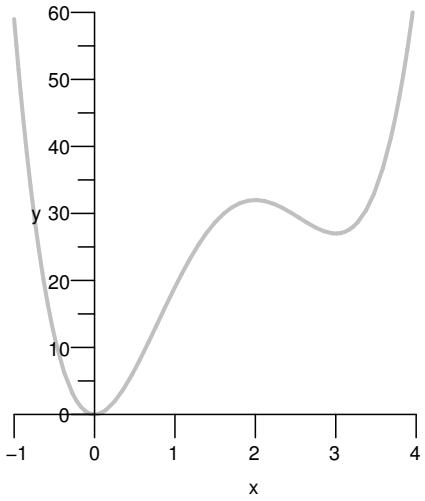
$$t(x) = -2x^3 + 15x^2 - 36x + 40 \quad \_\_\_\_\_$$



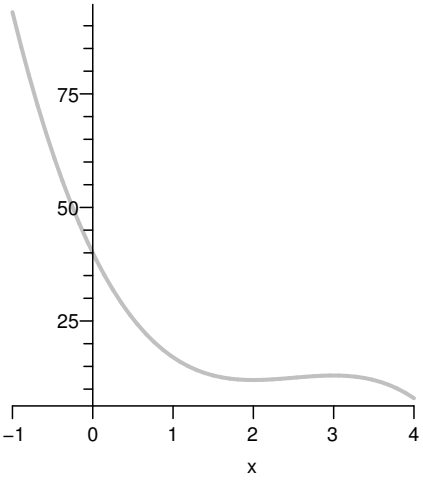
(a)



(b)



(c)



(d)

4. Evaluate the following limits. If you use L'Hôpital's rule, be sure to justify its use.

a. $\lim_{x \to 0} \dfrac{\sin x - x}{x^3}$

b. $\lim_{x \to 1} \left(\dfrac{1}{\ln x} - \dfrac{1}{x - 1} \right)$

c. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$

5. Evaluate the following limits *without using L'Hôpital's rule.*

a. $\lim_{x\to\infty}\left(\frac{x^3}{x^2+1}-\frac{x^2}{x+5}\right)$

b. $\lim_{x \to \infty} \dfrac{\sqrt{x^2+1}}{2x}$

6. Show that

$$\lim_{x \to -\infty} \sqrt{x^2+5x} - x = \infty.$$