

MATH 1311-1 FALL 2006

CALCULUS I

THIRD MIDTERM EXAM

FRIDAY, NOVEMBER 10, 9:30 AM - 10:20 AM

YOUR NAME (PLEASE PRINT):

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted.** You must justify all of your answers to receive credit.

You have **50 minutes** to work on all **6** problems. Please do all of your work on the paper provided.

The Honor Code requires that you neither give nor receive any aid on this exam.

FERPA waiver: By my signature I relinquish my FERPA rights in the following context: This exam paper may be returned en masse with others in the class and I acknowledge that I understand my score may be visible to others.

If I choose not to relinquish my FERPA rights, I understand that I will have to present my student ID at my instructor's office to retrieve my examination paper.

FERPA waiver signature: _____

Problem	1	2	3	4	5	6	Total
Points	15	15	20	15	15	20	100
Score							

Problem 1. [15 points] Find an equation of the line through the point $(2, 3)$ that cuts off the least area from the first quadrant. [*Hint:* Let x be the x -intercept of the line and let y be its y -intercept. Express the area cut off by the line in terms of x and y and then relate x and y by writing the slope of the line in two different ways.]

Problem 2. [3 points each] Compute the following limits (no partial credit is possible).

(a)

$$\lim_{x \rightarrow \infty} \frac{3x^2 - x + 4}{2x^2 + 5x - 8} =$$

(b)

$$\lim_{x \rightarrow -\infty} \frac{1 - x - x^3}{2x^2 - 7} =$$

(c)

$$\lim_{x \rightarrow \infty} \frac{1 + x^3 + x^5}{x - x^2 + x^4} =$$

(d)

$$\lim_{x \rightarrow -\infty} x^{3/5}(7 - x^{1/3}) =$$

(e)

$$\lim_{x \rightarrow \infty} \frac{x + 2}{\sqrt{9x^2 + 1}} =$$

Problem 3. [20 points] If

$$f(x) = \frac{x^2 + 4x + 4}{3x^2 + 12x + 21}$$

then

$$f'(x) = \frac{2x + 4}{(x^2 + 4x + 7)^2}$$

and

$$f''(x) = \frac{-6x^2 - 24x - 18}{(x^2 + 4x + 7)^3}.$$

Use this information to carefully sketch the graph of $y = f(x)$. Be sure to identify all roots, critical points, intervals of increase and decrease, local maxima and minima, intervals of concavity, inflection points, and vertical and horizontal asymptotes.

(Work Page)

Problem 4. [15 points] Let

$$g(x) = \frac{1}{x(x-1)}.$$

Compute the first 4 derivatives of g . [*Hint:* You may want to find numbers A and B so that $g(x) = A/(x-1) + B/x$.]

Problem 5. [15 points] Evaluate the following limits. If you need to use L'Hospital's rule, be sure to justify its use.

(a)

$$\lim_{x \rightarrow 1} \frac{1 - x + \ln x}{1 + \cos \pi x}$$

(b)

$$\lim_{t \rightarrow 0} \frac{e^{t/6} - e^{t/3}}{t}$$

(c)

$$\lim_{x \rightarrow 0} (1 - 2x)^{1/x}$$

Problem 6. [20 points] If

$$h(x) = x^{-2}e^x$$

then

$$h'(x) = x^{-3}(x - 2)e^{-x}$$

and

$$h''(x) = x^{-4}(x^2 - 4x + 6)e^x.$$

Use this information to carefully sketch the graph of $y = h(x)$. Be sure to identify all roots, critical points, intervals of increase and decrease, local maxima and minima, intervals of concavity, inflection points, and vertical and horizontal asymptotes.

(Work Page)