

MATH 1311 FALL 2007

CALCULUS I

THIRD MIDTERM EXAM

TUESDAY, NOVEMBER 13, 7:00 PM - 9:00 PM

YOUR NAME (PLEASE PRINT):

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted.** You must justify all of your answers to receive credit. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided.

The Honor Code requires that you neither give nor receive any aid on this exam.

If you are bound by the Academic Honor Code, please indicate that you have read and understood these guidelines by signing your name in the space provided:

Pledged: _____

Do not write below this line

Problem	1	2	3	4	5	6	7
Points	20	10	15	10	20	15	10
Score							

Total:_____

1. Evaluate the following definite integrals.

(a) $\int_{-3}^2 |t| \, dt$

(b) $\int_e^{e^e} \frac{1}{x \ln x} \, dx$

(c) $\int_0^1 y^3(1+y)^2 \, dy$

(d) $\int_1^{8/7} \frac{1}{x^2} \sqrt[3]{1 - \frac{1}{x}} \, dx$

2.

(a) Express the definite integral

$$\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$$

as a limit of Riemann sums. *Do not attempt to evaluate this limit.*

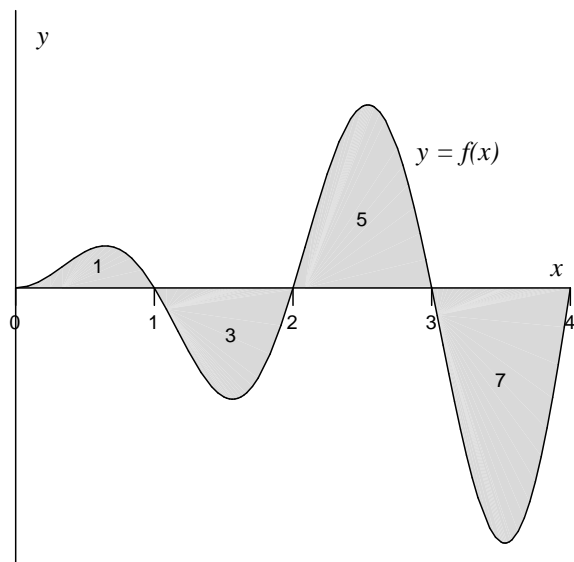
(b) Express the limit

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\sin \frac{i\pi}{n} \right) \frac{\pi}{n}$$

as a definite integral and evaluate using the Fundamental Theorem of Calculus.

3. Consider the function $f(x)$ whose graph on the interval $[0, 4]$ is shown below. The numbers in each shaded region indicate the area of that region. Let

$$g(x) = \int_1^x f(t) dt.$$



(a) Compute $g(0)$, $g(1)$, $g(2)$, $g(3)$ and $g(4)$.

(b) Find and classify the critical points of $g(x)$ in the interval $[0, 4]$.

- (c) Find the absolute maximum and minimum values of $g(x)$ on the interval $[0, 4]$.

4.

- (a) Verify that the function $G(x) = x \ln x - x$ is an antiderivative for $g(x) = \ln x$.

- (b) Evaluate

$$\int_1^{e^2} \ln x \, dx.$$

5. When its brakes are fully applied, a certain automobile has a constant deceleration of 22 ft/s^2 . If it is traveling at 90 mi/h when its brakes are applied, how long will it take to come to a stop? How many feet will the automobile travel during that time? Be sure to define all variable you use and specify their units! [*Note:* You might find it useful to know that 1 mi/h is exactly $22/15 \text{ ft/s}$.]

6. Sketch the region bounded by the curves $y = x^2 - x - 2$, $y = x + 1$, $x = -2$ and $x = 0$ and compute its area.

7. In a certain city the temperature (in °F) t hours after 9 A.M. was modeled by the function

$$T(t) = 50 + 14 \sin \frac{\pi t}{12}.$$

Find the average temperature during the period from 9 A.M. to 9 P.M.

