

## MATH 1311 FALL 2009

# CALCULUS I



# FINAL EXAM

FRIDAY, DECEMBER 11

YOUR NAME (PLEASE PRINT):

**Instructions:** This is a closed book, closed notes exam. **Use of calculators is not permitted.** Unless indicated otherwise, you must justify all of your answers to receive credit. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided.

The Honor Code requires that you neither give nor receive any aid on this exam.

Please indicate that you have read and understood these guidelines by signing your name in the space provided:

Pledged: \_\_\_\_\_

[illegible]

1. Evaluate the limit, or explain why it does not exist.

a.  $\lim_{t \rightarrow 0} \frac{1}{t}$

b.  $\lim_{x \rightarrow 1^-} \frac{|1 - x|}{x - 1}$

c.  $\lim_{x \rightarrow 1} \frac{1}{\ln x} \int_1^x \frac{1}{1 + t^3} dt$  [*Hint:* Use, with justification, L'Hôpital's Rule.]

d.  $\lim_{t \rightarrow 4} \frac{\sqrt{t} - 2}{t - 4}$

2. Find  $\frac{dy}{dx}$ .

a.  $y = \frac{x^3 + 3x}{x^2 - 5}$

b.  $xy^3 + x^3y = x - y$

**c.**  $y = \int_2^{\sin x} e^{-t^2} dt$

**d.**  $y = (e^{x^3} - 5)^7$

**3.** Evaluate the integral.

**a.**  $\int \tan \theta \, d\theta$

**b.**  $\int_0^4 \sqrt{5x} \, dx$

**c.**  $\int \frac{t}{(t+1)^3} \, dt$

**d.**  $\int_{-\pi/6}^{\pi/6} \cos 3x \, dx$

4.

a. Write down the limit definition of  $f'(x)$ .

b. If  $f(x) = x^2 - 3x$ , use the limit definition of the derivative to compute  $f'(x)$ .

**5.** Let  $q(x) = x^5 + 5x - 2$ .

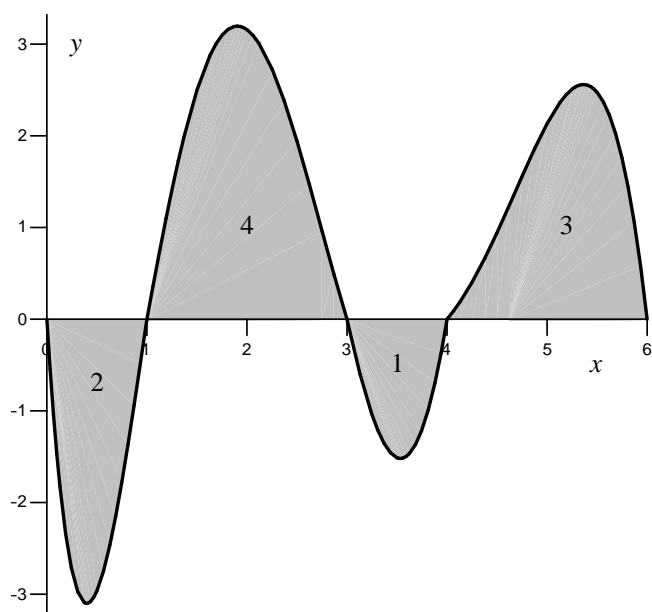
**a.** Explain why the equation  $q(x) = 0$  has at least one solution.

**b.** Explain why the equation  $q(x) = 0$  cannot have two solutions.

6. Use a linear approximation to estimate  $\ln 1.1$ .

7. The diagram below shows the graph of a function  $f(x)$ . The number in each shaded region is the area of that region. Let

$$g(x) = \int_0^x f(t) dt.$$



Graph of  $y = f(x)$ .

a. Find the intervals of increase and decrease for  $g$ .

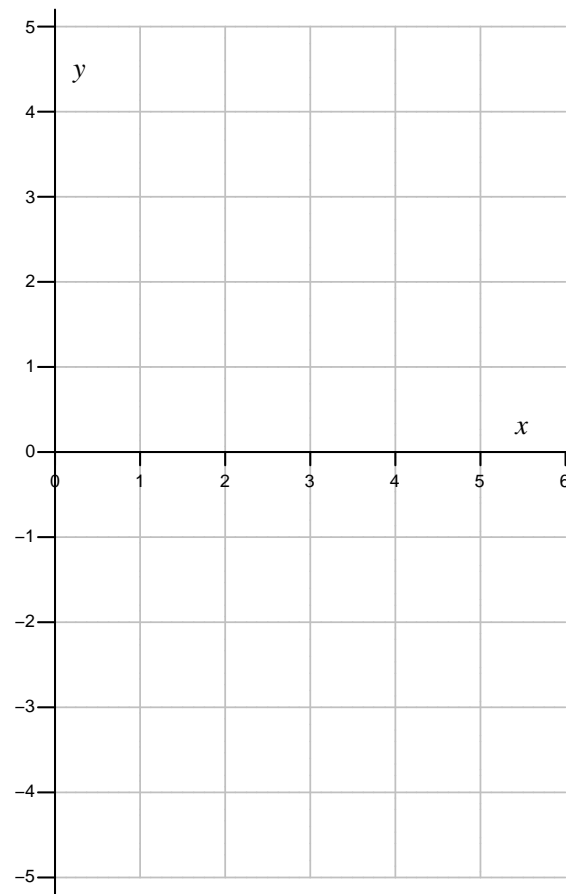


**b.** Find the local and absolute extreme values of  $g$ .

**c.** Find the (approximate) intervals of concavity for  $g$ .

**d.** Find the (approximate)  $x$ -coordinates of the inflection points of  $g$ .

e. Carefully sketch the graph of  $g(x)$ .



f. If  $h(x) = \int_1^x f(t) dt$ , how do the graphs of  $g$  and  $h$  differ?

8.

a. Show that  $G(x) = \ln |\sec x + \tan x|$  is an antiderivative of  $g(x) = \sec x$ .

b. Use part a to evaluate  $\int_0^{\pi/4} \sec x \, dx$ .

**9.** Two cars start moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h. At what rate is the distance between the cars increasing two hours later? [*Hint:* Your answer should be a whole number.]

**10.** A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

11. Let  $h$  and  $r$  be positive numbers and let  $L$  be the line through  $(0, h)$  and  $(r, 0)$ .

a. Find an equation for the line  $L$ .

b. If the region below  $L$  in the first quadrant is rotated about the  $y$ -axis, one obtains a circular cone with height  $h$  and radius  $r$ . Use Calculus to derive the familiar formula for the volume of this cone.



