

MATH 1311 SPRING 2008

CALCULUS I

FINAL EXAM

SATURDAY, MAY 10, 6:30 PM - 9:30 PM

YOUR NAME (PLEASE PRINT):

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted.** You must justify all of your answers to receive credit. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided.

The Honor Code requires that you neither give nor receive any aid on this exam.

If you are bound by the Academic Honor Code, please indicate that you have read and understood these guidelines by signing your name in the space provided:

Pledged: _____

Do not write below this line

Problem	1	2	3	4	5	6
Points	16	16	16	12	10	10
Score						

Problem	7	8	9	10	11	Total
Points	20	10	10	15	15	150
Score						

1. Evaluate the following limits.

(a) $\lim_{x \rightarrow -1} \frac{x+1}{x^2-1}$

(b) $\lim_{x \rightarrow \infty} \frac{7x^{13} + 12x^7 - 3x + 1}{5x^{13} - 8x^{11} + 2x^8 - x}$

(c) $\lim_{x \rightarrow 5^+} \frac{x+3}{x^2-3x-10}$

(d) $\lim_{x \rightarrow 3} \frac{e^{x-3} - 1}{\sin(x-3)}$

2. Find dy/dx .

(a) $y = \frac{1-x}{1+x}$

(b) $y = (x^3 + 2x + 1)e^{3x}$

(c) $y^2 - x^2 y + 2x^2 = 5$

(d) $y = \sqrt{\cos x}$

3. Evaluate the following integrals.

(a) $\int_0^1 y(1-y^3) dy$

(b) $\int \sqrt{\tan \theta} \sec^2 \theta \, d\theta$

(c) $\int_1^{e^\pi} \frac{\cos(\ln t)}{t} \, dt$

(d) $\int \frac{x}{\sqrt{2x+1}} \, dx$

4. Match each definite integral in (a) - (d) with one of the limits in (i) - (iv).

(a) $\int_0^1 \sin(\pi t^2) dt = \underline{\hspace{2cm}}$

(b) $\int_1^2 \sin(\pi x^2) dx = \underline{\hspace{2cm}}$

(c) $\int_0^\pi \sin(\theta) d\theta = \underline{\hspace{2cm}}$

(d) $\int_{-1}^1 \sin(\pi y) dy = \underline{\hspace{2cm}}$

(i) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \sin\left(\frac{\pi i}{n}\right) \frac{\pi}{n}$

(ii) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \sin\left(\pi \left(1 + \frac{i}{n}\right)^2\right) \frac{1}{n}$

(iii) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \sin\left(-\pi + \frac{2\pi i}{n}\right) \frac{2}{n}$

(iv) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \sin\left(\frac{\pi i^2}{n^2}\right) \frac{1}{n}$

5. For $0 \leq t \leq 3$, a particle travels along the x -axis with acceleration given by $a(t) = 2t + 4$. The particle is initially at rest, and ends up at the position $x = 0$.

(a) At what position was the particle initially?

(b) Find the average velocity of the particle during the interval $0 \leq t \leq 3$.

6. Sketch the region enclosed by the curves $y = -3x$ and $y = 4 - x^2$ and compute its area.

7. The graph of the function $f(x)$ consists of line segments and a semicircle, as shown below. For $0 \leq x \leq 7$ define

$$g(x) = \int_0^x f(t) dt$$

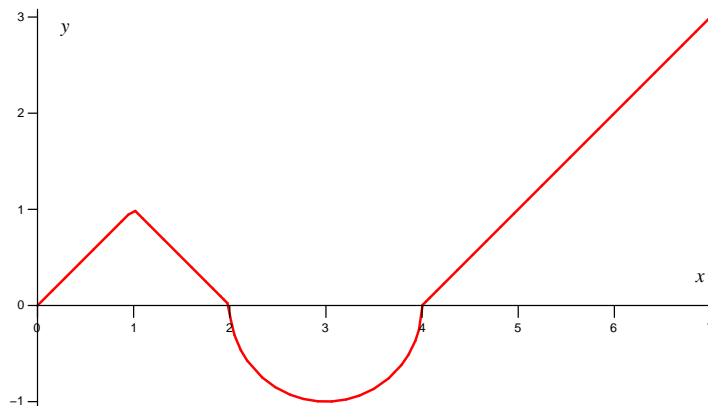


Figure 1: The graph of $y = f(x)$.

(a) Compute $g(0)$, $g(2)$, $g(4)$, $g(6)$ and $g(7)$.

(b) Show that there is a number c in $[2, 4]$ so that $g(c) = 0$.

(c) Find and classify the critical points of $g(x)$.

(d) Using the guidelines learned in class, carefully sketch the graph of $g(x)$ on the interval $[0, 7]$.

8.

- (a) Suppose $f(x)$ is continuous and the curve $y = f(x)$ passes through the points $(1, 2)$ and $(3, 5)$. Find the value of $\int_1^3 f'(x) dx$.

- (b) If $\int_0^1 g(x) dx = 10$, find the value of $\int_{-1/2}^0 g(2x + 1) dx$.

- (c) If $h(x)$ is continuous, find the derivative of the function

$$H(x) = x \int_5^{2x} h(t) dt.$$

9. Find all lines tangent to the graph of $y = 2x^3 - 3x^2 - 6x + 1$ which have slope equal to 6. Express your answers in the form $ax + by = c$.

10. A ladder 41 ft long that was leaning against a vertical wall begins to slip. Its top slides down the wall while its bottom moves along the level ground at a constant speed of 4 ft/s. How fast is the top of the ladder moving when it is 9 ft above the ground?

11. You must construct a closed rectangular box with volume 576 in.^3 and with its bottom twice as long as it is wide. Find the dimensions of the box that will minimize its total surface area.

