

MATH 1311 FALL 2007

CALCULUS I

FINAL EXAM

MONDAY, DECEMBER 10, 6:30 PM - 9:30 PM

YOUR NAME (PLEASE PRINT):

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted.** You must justify all of your answers to receive credit. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided.

The Honor Code requires that you neither give nor receive any aid on this exam.

If you are bound by the Academic Honor Code, please indicate that you have read and understood these guidelines by signing your name in the space provided:

Pledged: _____

Do not write below this line

Problem	1	2	3	4	5	6	7
Points	10	12	16	16	16	15	20
Score							

Problem	8	9	10	11	12	13	14
Points	10	10	15	15	15	15	15
Score							

Total:_____

1.

(a) Given a function $f(x)$, write down the limit definition of $f'(x)$.

(b) If $f(x) = x^2 - 2x$, use the limit definition of the derivative to compute $f'(x)$. Do not use l'Hôpital's rule to evaluate the limit!

2. Fill in the blanks. You do not need to justify your response.

The Fundamental Theorem of Calculus. Let $f(x)$ be _____ on the interval $[a, b]$.

(a) **Part 1.** If we define

$$F(x) = \int_a^x f(t) dt$$

then _____ on the interval $[a, b]$.

(b) **Part 2.** If $G(x)$ is any antiderivative for $f(x)$ on $[a, b]$ then

_____.

3. Evaluate the following limits.

(a) $\lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x - 7}$

(b) $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x + 1}$

(c) $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta \sin \theta}$

(d) $\lim_{t \rightarrow 0^+} t \ln t$

4. Find dy/dx .

(a) $y = (x^3 + 2x + 1)^2$

(b) $y = \frac{x}{1 + \cos x}$

(c) $y = (x+1)e^{x^2+3x}$

(d) $y = |x + 1|$

5. Evaluate the following definite integrals.

(a) $\int_0^1 x+2\sqrt{x}+3\sqrt[3]{x} \, dx$

(b) $\int_{-\pi/4}^0 \tan \theta \, d\theta$ $\left[\textit{Hint: } \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$

(c) $\int_1^4 \sqrt{x} e^{x\sqrt{x}} \, dx$

(d) $\int_2^3 x\sqrt{x-2} \, dx$

6. Find the absolute maximum and minimum values of the function $h(x) = 3x^{2/3} - x$ on the interval $[-8, 1]$.

7. Two ships are sailing toward a very small island. One ship, the *Arbitrary*, is east of the island and is sailing due west at 15 mi/h. The other ship, the *Beyond Doubt*, is north of the island and is sailing due south at 20 mi/h. At a certain time the *Arbitrary* is 30 mi from the island and the *Beyond Doubt* is 40 mi from the island. At what rate is the distance between the two ships changing at that time?

8. Find an equation for the tangent line to the curve $x^2 + xy + y^2 = 7$ at the point $(3, -2)$. Write your answer in the form $ax + by = c$.

9. Find the area of the surface obtained by revolving the part of the curve $y = x^3/3$ from $x = 0$ to $x = 1$ about the x -axis.

10. The base of a certain solid is the region in the first quadrant of the xy -plane bounded by the graph of the equations $x = y^2$ and $x = 2$. Every cross section of the solid perpendicular to the x -axis is a semicircle with its diameter in the xy -plane. Find the volume of this solid.

11. The region bounded by the curves $y = 2x^2$ and $y = x^2 + 4$ is revolved around the y -axis to produce a solid object. Compute the volume of this solid.

12. A cylindrical tank of radius 10 ft and height 20 ft is mounted vertically 50 ft above ground. Compute the amount of work done in filling the tank with a fluid of density ρ ft³/lb pumped in from ground level. [*Note:* Your final answer will be an expression involving ρ .]

13. The acceleration of a particle moving in a straight line is given by $a(t) = -2t - 1$. If its velocity at $t = 0$ is 6, determine the total distance the particle travels from $t = 0$ to $t = 3$.

14. A particle moves in a straight line along the x -axis subject to the force

$$F(x) = \frac{x^2 - 4}{x^2 + 3}$$

at each point.

- (a) If the particle begins at $x = 0$ and stops at $x = b$, write down an integral that expresses the total work done by the force acting on the particle. *Do not evaluate this integral.*

- (b) Determine where the particle should stop in order to minimize the total amount of work done by the force.

