

MATH 1311 FALL 2015

CALCULUS I

FINAL EXAM

FRIDAY, DECEMBER 11

YOUR NAME (PLEASE PRINT):

Instructions: This is a closed book, closed notes exam. **Use of calculators or other electronic devices such as cell phones, mp3 players, etc. is not permitted.** Unless indicated otherwise, you must justify all of your answers to receive credit. Unjustified answers and/or disorganized or otherwise illegible work will receive partial credit at best. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided.

The Honor Code requires that you neither give nor receive any aid on this exam.

Please indicate that you have read and understood these guidelines by signing your name in the space provided:

Pledged: _____

Do not write below this line

Problem	1	2	3	4	5	6	7
Points	16	16	16	10	4	5	10
Score							

Problem	8	9	10	11	12	13	14
Points	10	10	12	7	10	6	18
Score							

Total: _____

1. Evaluate the limit, if it exists.

a. $\lim_{t \rightarrow 0} \frac{\sqrt{1+t} - \sqrt{1-t}}{t}$

b. $\lim_{x \rightarrow 4^+} \frac{x^2 - 4x - 1}{x^2 - 3x - 4}$

c. $\lim_{y \rightarrow 0} \frac{e^y - 1}{\sin(2y)}$

d. $\lim_{x \rightarrow -\infty} \frac{3x - 5}{\sqrt{2x^2 + 1}}$

2. Find $\frac{dy}{dx}$.

a. $y = \frac{x^3 + 3 \cos(x)}{x^2 - 5}$

b. $xy^3 + x^3y = x - y$

c. $y = \int_2^{\sin x} e^{-t^2} dt$

d. $y = (e^{x^3} - 5)^7$

3. Evaluate the integral.

a. $\int e^{\tan x} \sec^2 x dx$

b. $\int_1^9 \frac{x-1}{\sqrt{x}} dx$

c. $\int_e^{e^4} \frac{dx}{x(\ln x)^2}$

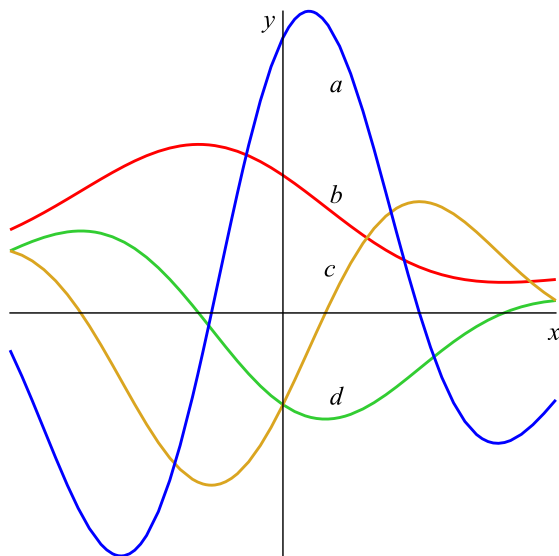
d. $\int x\sqrt{3x+2} dx$

4.

a. Write down the limit definition of $f'(x)$.

b. If $f(x) = 3x - x^2$, use the limit definition of the derivative to compute $f'(x)$. Do not use l'Hospital's rule!

5. The figure below shows the graphs of f , f' , f'' and f''' . Identify each curve. You do not have to justify your answers.



f	f'	f''	f'''

6. If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, how small can $f(4)$ possibly be? [*Suggestion:* Use the Mean Value Theorem]

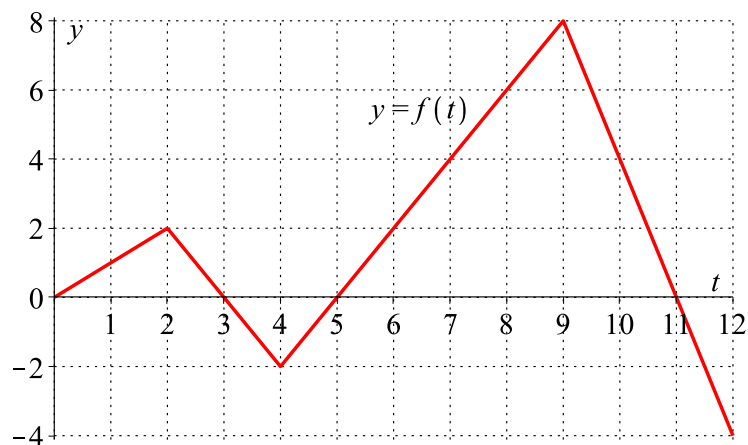
7. The tangent line to the graph of $y = f(x)$ at $x = 2$ is $2x + y = 3$. Use this to find an equation for the tangent line to $y = [f(x)]^2$ at $x = 2$.

8. Use a linear approximation to estimate $\sqrt[4]{15.9}$.

9. Two cars start moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h. At what rate is the distance between the cars increasing two hours later? Be sure to define any variables you introduce by using a diagram or otherwise. [*Hint:* Your answer should be a whole number.]

10. Consider the function $f(t)$ whose graph on the interval $[0, 12]$ is shown below. Let

$$g(x) = \int_3^x f(t) dt.$$



- a. Find the intervals on which g is increasing or decreasing.
- b. Find the x -coordinates of the local extrema of g , and determine the value of g at each of these points. Be sure to specify the type of extremum.

- c. Find the absolute maximum and minimum values of g on $[0, 12]$

11.

- a. Show that $F(x) = (x - 1)e^x$ is an antiderivative of $f(x) = xe^x$.

- b. Use part a to evaluate $\int_1^2 xe^x dx$.

12. Find the dimensions of a rectangle with area 1000 m^2 whose perimeter is as small as possible. Be sure to define any variables you introduce by using a diagram or otherwise.

13. Express

$$\lim_{n \rightarrow \infty} \frac{\pi}{n} \sum_{i=1}^n \cos \left(\frac{\pi}{2} + \frac{i\pi}{n} \right)$$

as a definite integral and then evaluate the integral.

14. Let R denote the region enclosed by the curves $y = 5x - x^2$ and $y = x$.

- a.** Carefully sketch the region R . Be sure to indicate where the two bounding curves cross each other.

b. Find the area of R .

c. If the region R is rotated about the x -axis, write down an integral that will compute the volume of the resulting solid. *Do not evaluate this integral.*