ALGEBRA II
FALL 2017

ASSIGNMENT 10
DUE NOVEMBER 8

Exercise 1. A *derivation* on a domain R is an additive homomorphism $d : R \to R$ satisfying

$$d(rs) = rd(s) + sd(r)$$

for all $r, s \in R$. Let d be a derivation on $\mathbb{Q}[x]$.

a. Show that $d(1) = 0$. Conclude that $d(n) = 0$ for all $n \in \mathbb{Z}$.

b. Show that $d(r) = 0$ for all $r \in \mathbb{Q}$.

c. Show that $d(f) = f' \cdot d(x)$ for all $f \in \mathbb{Q}[x]$.

d. Let $h \in \mathbb{Q}[x]$. Show that $d(f) = hf'$ defines a derivation on $\mathbb{Q}[x]$. Together with part **c**, this classifies all the derivations of $\mathbb{Q}[x]$.

Exercise 2. Let F be a field. Show that the roots of $x^n - 1$ in F form a subgroup of $F^\times$.