Algebra II
Fall 2017

Assignment 11.1
Due November 15

Exercise 1. Let F be a field of characteristic $p \neq 0$ and $f \in F[x]$ irreducible. Prove that there exist an irreducible $g \in F[x]$ with no repeated roots and a $k \in \mathbb{N}_0$ so that $f(x) = g(x^{p^k})$. Use this to factor f in its splitting field over F.

Exercise 2. Let $f(x) = x^p - x + 1 \in \mathbb{F}_p[x]$.

a. Show that f has no roots in $\mathbb{F}_p$.

b. Let α be a root of f in some extension of $\mathbb{F}_p$. Show that the set of roots of f is $\{\alpha + a \mid a \in \mathbb{F}_p\}$. Hence the splitting field of f is $K = \mathbb{F}_p(\alpha)$.

c. Show that f is irreducible over $\mathbb{F}_p$. [*Suggestion:* If $f = gh$ is any nontrivial factorization in $\mathbb{F}_p[x]$, split g over K and then compute its trace (degree minus one) coefficient to arrive at a contradiction.]