Algebra II
Fall 2017

Assignment 11.2
Due November 15

Exercise 1. Let K/F be a field and suppose $a, b, \in K$ are algebraic. Show that if a has degree m over F, b has degree n over F and m and n are relatively prime, then $[F(a,b) : F] = mn$.

Exercise 2. Let K/F be a finite extension of fields. If $f \in F[x]$ is irreducible and $[K:F]$ is relatively prime to $\deg f$, show that f is irreducible over K.

Exercise 3. Let p and q be primes. Find the degree of the splitting field of $x^p - q$ over $\mathbb{Q}$.

Exercise 4. Find the minimal polynomials of the following numbers over $\mathbb{Q}$.

a. $\sqrt[3]{2} + \sqrt[3]{4}$

b. $\sqrt{-3} + \sqrt{2}$

Exercise 5. Suppose that $[K : \mathbb{Q}] = 2$. Show that $K = \mathbb{Q}(\sqrt{n})$ for some $n \in \mathbb{Z}$ that is not divisible by the square of any prime (i.e. n is *square free*). Furthermore, show that n uniquely determines K.

Exercise 6. Let F be a field, $f \in F[x]$ and K be the splitting field for f over F. Suppose $g \in F[x]$ is irreducible and $g(a) = 0$ for some $a \in K$. Show that g splits in K. [*Suggestion:* Let b be any other root of g and $\sigma : F(a) \to F(b)$ the isomorphism over F carrying a to b. Use the "isomorphism lifting property of splitting fields" to show that $K \cong K(b)$ over F, and then argue that $K(b)$ has degree 1 over K as a result.]