Algebra II
Fall 2017

Assignment 12.1
Due December 4

Exercise 1. Let p be a prime number, $n \in \mathbb{N}$. Prove that $\mathbb{F}_{p^n}/\mathbb{F}_p$ is Galois and that its (automorphism) group is cyclic, generated by the *Frobenius automorphism* $\varphi(x) = x^p$. [*Suggestion:* Once you show φ is a field automorphism of $\mathbb{F}_{p^n}$ fixing $\mathbb{F}_p$, compute the order of φ and show that it agrees with the degree of the extension. Why does this suffice?]

Exercise 2. Let p and q be distinct primes. Show that $\mathbb{Q}(\sqrt{p}, \sqrt{q})/\mathbb{Q}$ is Galois. What is the Galois group? [*Suggestion:* Use the fact that if r is prime, then there are only two groups (up to isomorphism) of order r^2: $\mathbb{Z}/r\mathbb{Z} \times \mathbb{Z}/r\mathbb{Z}$ and $\mathbb{Z}/r^2\mathbb{Z}$.]

Exercise 3. Let F be a field, $f \in F[x]$ with roots $a_1, a_2, \ldots, a_n$ in some splitting field, and $K = F(a_1, a_2, \ldots, a_n)$. Prove that $G = \mathrm{Aut}_F(K)$ is isomorphic to a subgroup of S_n. [*Suggestion:* Consider the action of G on the set of roots of f.]