ALGEBRA II
FALL 2017

ASSIGNMENT 12.2
DUE DECEMBER 4

Exercise 1. Let G be a group with normal subgroup N.

- **a.** Let $\mathcal{H} \leq G/N$. Show that there is an H with $N \leq H \leq G$ so that $\mathcal{H} = H/N$.
- **b.** In the notation of part **a**, show that $H/N \lhd G/N$ if and only if $H \lhd G$ and that in this case

$$(G/N)/(H/N) \cong G/H.$$

Exercise 2. Let F be a perfect field and $f \in F[x]$. Prove that the splitting field of f is Galois over F.

Exercise 3. Let a be separable over F and $k = F(a)$. Prove that there is a minimal Galois extension K/F containing k, the *Galois closure of* k/F.

Exercise 4. Let K denote the splitting field of $x^3 - 2$ over $\mathbb{Q}$. Determine the Galois group $\mathrm{Aut}_{\mathbb{Q}}(K)$.