Algebra II
Fall 2017

Assignment 2.1
Due September 8

Exercise 1. Let p be a prime, $q(x) \in \mathbb{Z}_{p^2}[x]$ and $a \in \mathbb{Z}_{p^2}^{\times}$. Show that $a + p \cdot q(x)$ is a unit in $\mathbb{Z}_{p^2}[x]$. [*Suggestions:* See Exercise 2a of Assignment 1.2. Or use Exercise 3b.]

Exercise 2. Use the algorithm discussed in class to find the inverse of 139 in $\mathbb{Z}_{532}$.

Exercise 3. Let R be a commutative ring with unity and suppose $b \in R$ is nilpotent.

a. Show that $1 \pm b \in R^{\times}$. [*Suggestion:* Recall the sum of a geometric series from Calculus II: $\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$. Establish a similar identity in the current setting.]

b. If $a \in R^{\times}$, show that $a \pm b \in R^{\times}$. [*Suggestion:* Factor out a and use part **a**.]