Exercise 1. Let R be a commutative ring. Show that the subset of nonzero non-zero-divisors of R is closed under multiplication.

Exercise 2. Let R be a commutative ring with unity. Show that if R has the Cancellation Property, then R is a domain.

Exercise 3. Let F be a finite field with n elements. Prove that $\alpha^{n-1} = 1$ for all $\alpha \in F \setminus \{0\}$.

Exercise 4. Let R be a finite commutative ring with unity. Prove that every nonzero element of R is either a zero-divisor or a unit. Note that this implies every finite domain is a field. [*Suggestion:* Show that if $a \in R \setminus \{0\}$ is not a zero-divisor, the map $R \setminus \{0\} \to R \setminus \{0\}$ given by $x \mapsto ax$ is injective.]

Exercise 5. Let R be a ring with unity 1_R and consider the equation

$$x^2 + x - 2 = 0, \tag{1}$$

where $2 = 2 \cdot 1_R = 1_R + 1_R$.

- **a.** Show that if R is a domain, then (1) has at most two solutions. [*Suggestion:* Factor the polynomial.]
- **b.** Show that if $R = \mathbb{Z}_{10}$, then (1) has exactly 4 solutions.
- **c.** Show that if $R = \mathrm{M}_2(\mathbb{R})$, then (1) has infinitely many solutions. [*Suggestion:* Find one solution and then conjugate it by $\mathrm{GL}_2(\mathbb{R})$.]