Algebra II
Fall 2017

Assignment 3.1
Due September 13

Exercise 1. Let R be a ring with positive characteristic. Prove that if $m \in \mathbb{N}$ has the property that $mx = 0$ for all $x \in R$, then $\operatorname{char} R \mid m$. [*Suggestion:* Use the division algorithm.]

Exercise 2. Let R be a commutative ring with unity. Prove that $\operatorname{char} M_n(R) = \operatorname{char} R$.

Exercise 3. Let D be a domain and $P = \{n \cdot 1 \mid n \in \mathbb{Z}\}$ (i.e. P consists of all integral multiples of 1). Prove that P is a domain. What can you say about the order (number of elements of) of P?

Exercise 4. Let R be a commutative ring with unity and let $a_1, a_2, \ldots, a_n \in R$. Show that

$$\langle a_1, a_2, \ldots, a_n \rangle = Ra_1 + Ra_2 + \cdots + Ra_n = \{r_1 a_1 + r_2 a_2 + \cdots r_n a_n \mid r_i \in R\}$$

is an ideal of R, the *ideal generated by* $a_1, a_2, \ldots, a_n$.

Exercise 5. Let I be an interval in $\mathbb{R}$ and let $C(I)$ denote the ring of continuous functions on I. Given $S \subseteq I$, let $Z(S) = \{f \in C(I) \mid f(s) = 0 \text{ for all } s \in S\}$. Show that $Z(S)$ is an ideal in $C(I)$.