Algebra II
Fall 2017

Assignment 3.2
Due September 13

Exercise 1. Let A and B be ideals of a ring R. Prove that

$$A + B = \{a + b \,|\, a \in A, b \in B\}$$

is an ideal.

Exercise 2. Let R be a commutative ring with unity and let $a_1, a_2, \ldots, a_n \in R$.

a. Prove that

$$I = Ra_1 + Ra_2 + \cdots Ra_n = \{r_1a_1 + r_2a_2 + \cdots r_na_n \,|\, r_i \in R\}$$

is an ideal. [*Suggestion:* Use the preceding exercise.]

b. Show that if J is an ideal of R and $a_1, a_2, \ldots, a_n \in J$, then $I \subseteq J$.

Exercise 3. Let A and B be ideals of a ring R. Prove that

$$AB = \{a_1b_1 + a_2b_2 + \cdots a_nb_n \,|\, a_i \in A, b_i \in B, n \in \mathbb{N}\}$$

is an ideal.

Exercise 4. Let A and B be ideals in a ring R.

a. Prove that $A \cap B$ is an ideal.

b. Prove that $AB \subseteq A \cap B$.

c. Suppose R is commutative with unity. Prove that if $A + B = R$, then $A \cap B = AB$. [*Suggestion:* Write $1 = a + b$ with $a \in A$ and $b \in B$.]