Algebra II
Fall 2017

Assignment 4.1
Due September 20

Exercise 1. Let R be a commutative ring and let A be an ideal of R.

a. Show that the *nil radical* of A,

$$N(A) = \{r \in R \mid r^n \in A \text{ for some } n \in \mathbb{N}\},$$

is an ideal of R.

b. Show that $R/N(\{0\})$ has no nonzero nilpotent elements.

Exercise 2. Recall that for an interval $I \subseteq \mathbb{R}$, $C(I)$ denotes the ring of continuous (real-valued) functions on I. Let $S = \{r_1, r_2, \ldots, r_n\} \in I$ and

$$Z(S) = \{f \in C(I) \mid f(r_i) = 0 \text{ for all } i\}.$$

Recall that $Z(S)$ is an ideal of $C(I)$. Show that the map

$$C(I)/Z(S) \to \underbrace{\mathbb{R} \oplus \mathbb{R} \oplus \cdots \oplus \mathbb{R}}_{n \text{ times}}$$
$$f + Z(S) \mapsto (f(r_1), f(r_2), \ldots, f(r_n))$$

is a well-defined bijection. [*Suggestion:* For surjectivity, look into polynomial interpolation.]

Exercise 3. Show that $\mathbb{Z}[i]/\langle 2 - i\rangle$ is a field. How many elements does it have?