Algebra II
Fall 2017

Assignment 5.1
Due September 27

Exercise 1. Let $n \in \mathbb{N}$. Show that the map $\phi : \mathbb{Z} \to \mathbb{Z}_n$ given by $a \mapsto a \bmod n$, where $a \bmod n$ denotes the remainder when a is divided by n, is multiplicative.

Exercise 2. Let R and S be commutative rings with unities, $\phi : R \to S$ an epimorphism and $A \subseteq S$ an ideal.

- **a.** Show that if A is prime, then $\phi^{-1}(A)$ is prime.
- **b.** Show that if A is maximal, then $\phi^{-1}(A)$ is maximal.

Exercise 3. Let R and S be rings and $I \subseteq R$, $J \subseteq R$ ideals. Use the First Isomorphism Theorem to prove that

$$(R \oplus S)/(I \oplus J) \cong R/I \oplus S/J.$$