Algebra II
Fall 2017

Assignment 5.3
Due September 27

Exercise 1. Let F be a field, $a \in F$ and $f \in F[x]$. We say a is a *root* of f if $f(a) = 0$.

- **a.** Use the Division Algorithm to prove the *Factor Theorem*: a is a root of f if and only if $f = (x - a)g$ for some $g \in F[x]$.
- **b.** Use induction to show that if a is a root of f then there is a unique $m \in \mathbb{N}$ so that $f = (x - a)^m h$, where $h \in F[x]$ and $h(a) \neq 0$. The integer m is called the *multiplicity* of the root a.

Exercise 2. Let F be a field and $a \in F$. Use the "evaluation at a" map and the First Isomorphism Theorem to show that $F[x]/\langle x - a \rangle \cong F$. Conclude that $\langle x - a \rangle$ is a maximal ideal in $F[x]$.

Exercise 3. If R is a commutative ring and P is a prime ideal in R, show that $P[x]$ is a prime ideal in $R[x]$.